CALCULATING PROBABILITY-BASED DISCOUNTS FOR LACK OF MARKETABILITY

Liquidity represents the ability to sell an investment quickly when the investor decides to sell. Conversely, then, ***lack*** of liquidity represents the cost of failing to realize gains or failing to avoid losses on an investment during the period in which the investor is offering it for sale. With that understanding, DLOM should reflect the volatility of the value of the investment during the period of time that it is being marketed.

REPLACING THE TRADITIONAL VIEW OF LEVELS OF VALUE

The valuation profession has written volumes about "levels of value" over the years. And the thoughts of those writers have evolved into a traditional view of levels of value that has driven much of valuation theory and practice as theorists and practitioners alike have tried to find appropriate benchmarks and methodologies for determining the equivalent marketable and non-marketable values of privately held securities and other assets. The Traditional View is –

Strategic Value

Difference reflects synergies

Control Value

Difference reflects the value of control

Publicly Traded Value

Difference reflects the value of marketability

Non-Marketable Minority Value

The essential thoughts behind the Traditional View are premised on the notion that Publicly Traded Value represents a marketable minority value, that control is worth more than liquidity, and that strategic value is worth more than control. It is this author's contention, however, that the logic supporting the Traditional View is topsy-turvy, resulting in erroneous interpretations of empirical evidence of levels of value and, in turn, to erroneous valuation conclusions. For example, the conventional wisdom has been that the "control premium" regularly measured by MergerStat is proof that Control Value represents a higher level of value than Publicly Traded Value – all other things equal. But does the Traditional View hold if the interpretation given to MergerStat's "control premium" is incorrect, and that it instead measures the discount (or a portion of the discount) imposed on poorly run public companies? Another example of potentially faulty Traditional View logic is the notion that Publicly Traded Value exclusively represents the return expectations of minority stakeholders. But does the Traditional View hold if instead the returns realized on publicly-traded securities represent "exchange rates" at which the expectations of controlling and minority interests are indifferent?

When comparing the relative values of controlling and minority interests in the same privately-held company, it is easy to intuit that the ability to control the enterprise is worth more than not having that ability. Hence, all other things equal, Control Value is logically greater than Minority Value. But that logic does not lead to a conclusion that Control Value is greater than Publicly Traded Value on a per share basis. For example, imagine a controlling interest in a publicly traded company. The controlling investor is exposed to the same price volatility as the minority investors, but is denied the opportunity to make a quickly dispose of his interest in the company. This realization suggests that liquidity (because it offers the ability to protect the value of one's investment) is worth more than control.

Let us explore the factors that result in different levels of value. When comparing the value drivers of well run publicly traded and well run privately controlled businesses, we find that the only real difference is liquidity or its lack:

Public Companies	**Private Companies**
Earnings / Cash Flow	Earnings / Cash Flow
Growth	Growth
Industry Risk	Industry Risk
Size Risk	Size Risk
Market Fluctuations	Market Fluctuations
Liquidity	**No Liquidity**

Liquidity represents the ability to sell an investment quickly when the investor decides to sell in order to lock in gains or to avoid losses. With that understanding, assuming everything else to be equal, the inability to quickly liquidate a controlling interest in a publicly traded company suggests that it is worth less per share than the liquid minority shares.

Some authors recently have been suggesting a Modified View wherein control value and publicly traded value may be very close to the same. Nevertheless, it is this author's opinion that both the Traditional and Modified Views of levels of value are incorrect. On a condensed basis, it is this author's opinion that levels of business value should be viewed this way –

Publicly Traded Value

Difference reflects the economic risk of lack of marketability

Non-Marketable Control Value

Difference reflects the economic risk of lack of control

Non-Marketable Minority Value

The basis of this Restructured View is straightforward. First, the investment returns of publicly traded companies should be viewed as "public company returns" not as "marketable minority returns." For well run companies that are operating optimally for their shareholders, there should be no economic difference between public company operating results and operating results to controlling interests of otherwise identical private companies – the material perquisites of control have been squeezed out of the public companies. If this were not essentially true, then publicly traded companies would not be able to attract capital in the form of fractional ownership. And, in fact, poorly run companies (i.e. those not operating optimally for their shareholders) have difficulty maintaining shareholder value and raising new capital.

Second, strategic value does not enter into the determination of required rates of return. Instead, the benefits of strategic acquisitions are shared throughout the surviving company as revenues are enhanced and expenses are minimized. Such effects are reflected in the income statement and cash flow of the enterprise as a whole and contribute to increased value that is shared by all ownership interests. Furthermore, such effects are not suggestive of the notion that Strategic Value is worth more than Publicly Traded Value. Although a value may be derived from a strategic opportunity that does not suggest that the opportunity is worth more than value of liquidity once the opportunity is realized. After all, once the opportunity is realized the new owners are subject to the same price volatility as the owners of publicly traded securities.

There are well run publicly traded companies and well run privately held companies. There are also poorly run companies of both types. When a public company is acquired at a premium above its publicly traded value it is a reflection of the perception that the acquired company is not maximizing its economic opportunities and shareholder value. Well-run publicly traded companies (i.e. those that are maximizing their economic opportunities and shareholder value) are not taken private – they are too expensive. Accordingly, the "premium" observed when publicly traded companies are taken private reflects the anticipation that inefficiencies in the acquired company can and will be eliminated. For these reasons, the so-called "control premium studies" are misused when used to suggest that control is worth more than liquidity.

Consider these thoughts: (1) Risk adjusted rates of return are fungible.[1] (2) There is a transaction cost to becoming and continuing as a publicly traded company. This creates a disincentive that can only be justified by (a) greater access to capital, and (b) the "pop" in value that the pre-IPO owners receive when their business goes public. (3) If control were worth more than liquidity, then the owners of privately held businesses would have a further disincentive to going public. (4) If control were more valuable than liquidity, then there would be no public companies.[2] (5) If control were worth more than liquidity, then large private equity firms such as

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[1] See Eric W. Nath, ASA, and M. Mark Lee, CFA "Acquisition Premium High Jinks," 2003 International Appraisal Conference, American Society of Appraisers; Eric W. Nath, ASA, "How Public Guideline Companies Represent 'Control' Value for a Private Company," Business Valuation Review, Vol. 16, No. 4, December 1997; and Eric W. Nath, "Control Premiums and Minority Discounts in Private Companies," Business Valuation Review, Vol. 9, No. 2, June 1990.

[2] Id.

Blackstone and KKR would not be converting to publicly traded companies. Thus it seems counter-intuitive that control should be viewed as equal in value to – or even more valuable than – liquidity.

Under otherwise identical circumstances, any given investment should have a greater value if it is immediately marketable than if it is not. Why is this so? Because liquidity allows the investor to avoid the economic risks of illiquidity. The notion of a control premium vis-à-vis public company values is illogical. Such premiums mathematically equate to lower rates of return. But since it is expected that it would take longer to sell a controlling interest in an optimally run private company than an interest in an otherwise identical public company, the required rate of return of the private company investor should be greater, not lower, than that of the public company investor. Thus, private company values should reflect a discount, not a premium, relative to comparable public company values.

Figure 1

ALTERNATIVE VIEW OF THE LEVELS OF BUSINESS VALUE

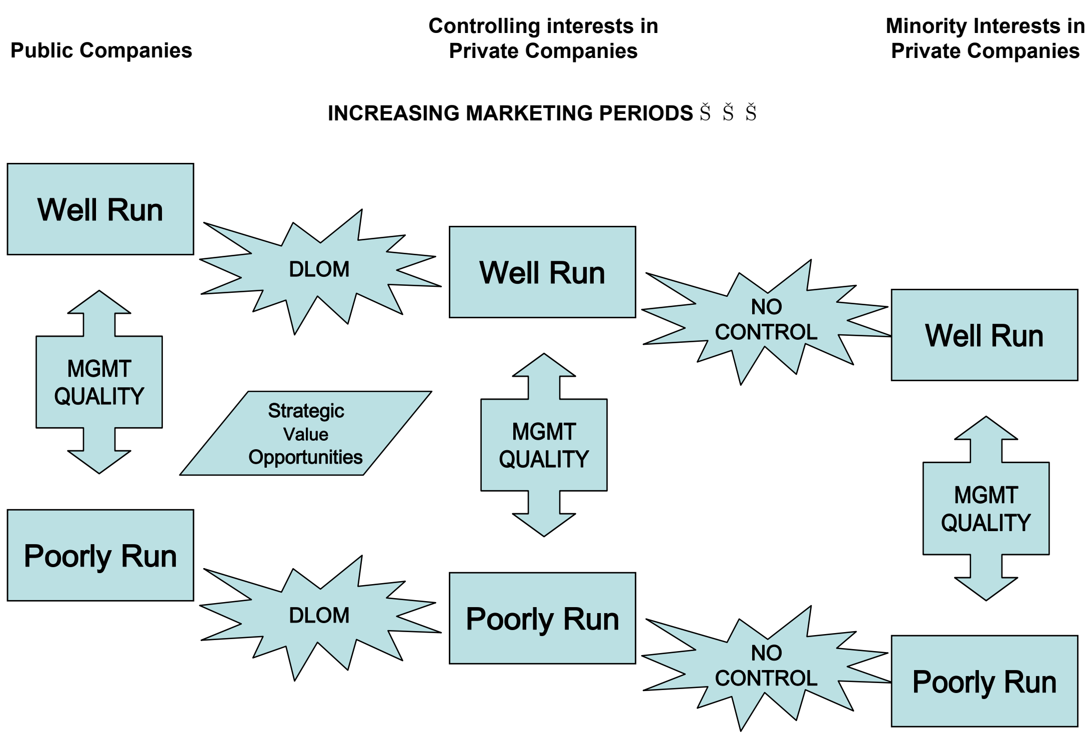

Figure 1 presents my alternative view of the relative levels of business investment value in greater dimension. The depiction shows how well run and poorly run private companies relate to each other and how the opportunity to realize strategic value (including synergy) arises from the conversion of poorly run firms into firms that hopefully will be well run. The depiction also demonstrates that all privately held companies – even controlling interests – are subject to the cost of illiquidity.[3] Even assuming all other things being equal, it simply takes longer to sell a controlling interest in a privately held business than it takes to sell an interest in a publicly traded company. Minority interests in privately held companies are worth less than controlling interests for two reasons: (1) such minorities generally lack the ability of controlling owners have to realize the perquisites of ownership and (2) the economic risks of lack of control result in longer periods of time to sell minority interests than it takes to sell the controlling interest in the same private company.

HOW THE EMPIRICAL STUDIES OF DISCOUNTS AND LIQUIDITY RELATE TO EACH OTHER

Conventional business valuation has used the well-publicized results of restricted stock studies, pre-IPO studies, and registered versus unregistered stock studies to effectively guess at appropriate DLOM percentages to use in their valuation reports. Understandably, such subjective means of applying the traditional approaches have been broadly unsatisfactory to the valuation community and the courts.

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[3] It has been suggested by some practitioners that discounts for lack of liquidity should not be applied to controlling interests because the earnings and cash flow of the company offset the discount while it is being held for sale. This argument fails because (1) it relies on a flawed view of the levels of value that ignores the facts that (a) rates of return derive from analysis of publicly traded stocks, and (b) liquidity is the only driver of value of publicly traded companies not present in privately held companies; (2) the economic circumstance of holding period earnings and cash flow also exists for minority interests; and (3) the holding period earnings and cash flow of both controlling interest and minority interest investments are necessarily already included in the capitalized or discounted values of the investments.

Figure 2

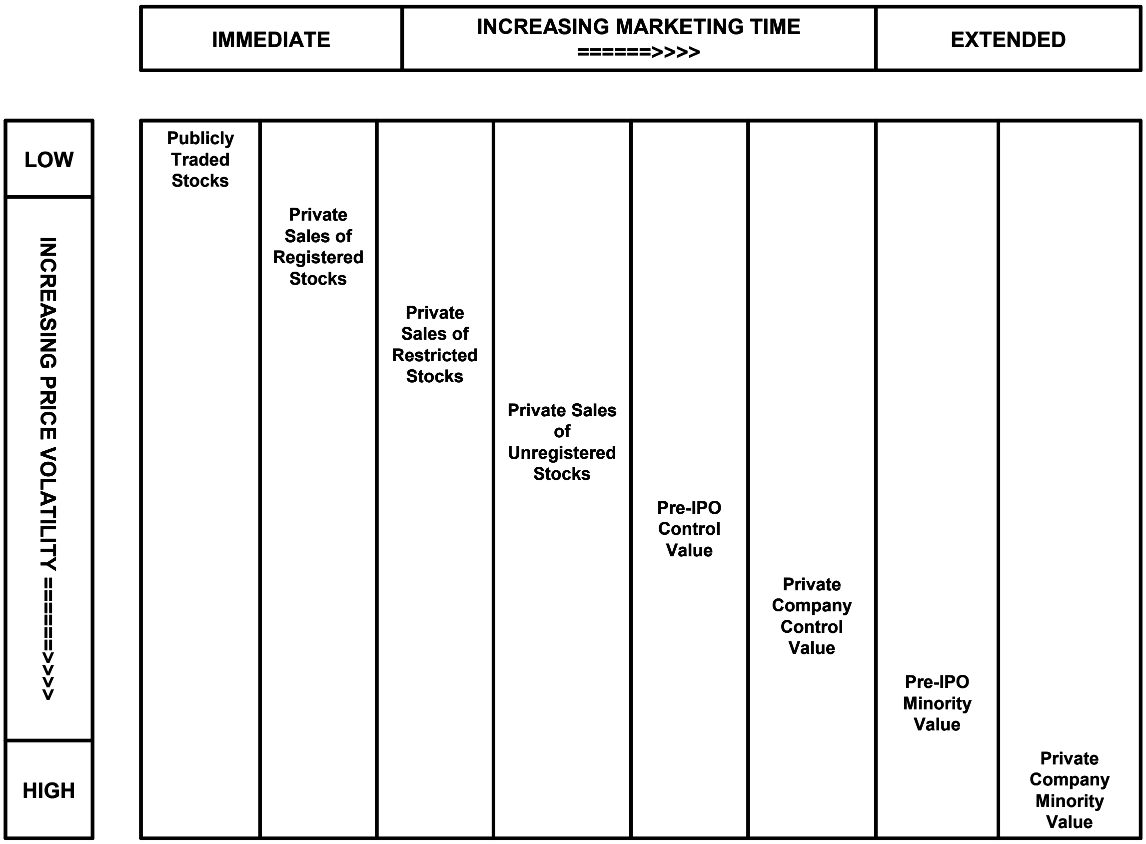

Figure 2 presents a relational stratification of the types of empirical studies that researchers have performed to explore the cost of illiquidity. I have attempted to present the studies in relative position based on marketing time and volatility assuming all other aspects of investment were equal. The presentation is instructive in enhancing understanding of what the various studies are measuring, how they relate to publicly traded values, and the extent to which they meet the needs of business valuation.

- Publicly traded companies are the standard against which all of the studies measure results and from which rates of return are calculated. Interests in publicly traded companies are worth more than interests in identical privately held companies because they can be sold immediately to realize gains and to avoid losses. Interests in privately held companies cannot.
- Private sales of publicly registered stocks typically involve large blocks of stock that could be sold into the public marketplace, but which would materially adversely affect stock prices if the entire block were to be dumped into the market at once. Avoiding that effect results in an extended period of time to liquidate the investment position in the public market during which time the

investor is subject to market risk. Negotiating a private sale of the block can accelerate liquidating the position, but the need to find a buyer with the wherewithal to purchase the block restricts the number of potential buyers and represents a diminution of demand for the stock. Although private sales of large blocks of registered stocks can somewhat mitigate the market risk, the risk does not go away. The buyer of the block assumes the risks, in turn, of having to sell into a limited pool of buyers or slowly feeding the block into the public market. These risks require compensation by means of a discount (i.e. DLOM).

- Private sales of restricted stocks in public companies have the same price risks as private sales of large blocks of registered stocks, but have the additional risk of being locked out of the public market for specific periods of time or being subject to restrictive "dribble out" rules. Accordingly, restricted stocks often can only be sold quickly in private sale transactions, which take longer than it does to sell unrestricted stocks in the public market.[4] The result is that a restricted registered stock is worth less than an unrestricted stock in the same company because of the greater market risk associated with the extended marketing period.
- Private sales of unregistered stocks in public companies typically involve large blocks of stock. They are worth less than equivalent blocks of registered stock (whether restricted or unrestricted) in the same publicly traded company because there is a cost to ultimate registration of the stock that further restricts the potential number of buyers of the block.[5] This results in relatively greater uncertainty, a relatively longer time to market the interest, and a relatively greater exposure to the risks of the marketplace.
- Pre-IPO private sales of controlling interests should have relatively longer marketing periods than for private sales of unregistered stocks in public companies, because the fact and timing of the IPO event can be uncertain. Furthermore, low pre-IPO stock sales prices may reflect compensation for

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[4] Some restricted stocks cannot be sold at all for contractually determined periods of time. Such investments have even greater economic risks than those merely subject to the "dribble out" rules.

[5] This discount is considered by Mukesh Bajaj, David J. Dennis, Stephen P. Ferris and Atulya Sarin in their paper "Firm Value and Marketability Discounts." Their study isolates the value of liquidity by comparing the stock sales of 88 companies that had sold both registered and unregistered stock private offerings. This approach does not, however, address the discount applicable to the additional time it takes to sell controlling or minority interests in private companies. Instead, it measures the value of stock registration. See Section IV.C of "Firm Value and Marketability Discounts."

services rendered. I am not aware of any studies that specifically address discounts observed in sales of controlling interests in pre-IPO companies.

- Private sales of controlling interests in a company that has no expectation of going public should be worth less than an otherwise identical company with an anticipated IPO event. Uncertain or not, an anticipated IPO event should result in a shorter marketing period than not anticipating such an event.
- Pre-IPO sales of non-controlling interests in a company planning an IPO event should be worth less than the controlling interest in the same company even without the planned IPO. The inability to control whether the planned IPO goes forward should result in greater uncertainty and a longer marketing period to liquidate the investment than would be experienced by the controlling investor. Also, low pre-IPO share prices may reflect compensation for services rendered.
- Non-controlling interests in private companies require greater discounts than all of the preceding circumstances because the relative risks of lacking control cause the period of time to liquidate the position to be potentially much longer than for the controlling interest in the same company or for otherwise comparable minority positions in firms with a planned IPO event.

WHY THE EMPIRICAL STUDIES ARE INADEQUATE FOR ESTIMATING DLOM

Restricted stock studies and pre-initial public offering ("pre-IPO") studies have been used to quantify discounts for lack of marketability ("DLOM") since the early 1970s. Despite making a good case for the need for a DLOM when valuing an investment that is not immediately marketable, this article will demonstrate that the study results are unreliable for calculating the DLOM applicable to a particular valuation engagement.

Unfortunately, the empirical studies of marketability discounts have limited utility to the appraiser opining on the fair market value of a business interest. Several authors have noted that most publicly traded firms do not issue restricted stock. This dearth necessitates samples of limited sizes, in limited industries, with data spread over long periods of time. The result has been substantial standard errors in their estimates.

The restricted stock studies measure the difference in value between a publicly traded stock with and without a time restriction on sale. Left unanswered is whether there is a difference between the restricted stock value of a publicly traded company and the value of that company if it were not publicly traded at all.

The pre-IPO studies reflect substantial standard errors in their estimates for similar reasons, but are also distorted by the facts that the studies necessarily are limited to successful IPOs; there are no post-IPO stock prices for failed IPOs. The discounts observed in the pre-IPO studies may also reflect uncertainty about whether the IPO event will actually occur, when the IPO event will occur, at what price the event will occur, and compensation for services rendered.

It should also be noted that all of the companies in the restricted stock and pre-IPO studies are, in fact, publicly traded. But essentially none of the privately held companies that are the subject of business valuations have a foreseeable expectation of ever going public. Accordingly, the circumstances of the privately held companies are highly distinguishable from those of the publicly traded companies that are the subjects of the studies. Thus, the pre-IPO studies are of dubious value for determining the DLOM of privately held companies.

Bajaj, et al., studied the difference in value observed when comparing private sales of registered stocks with private sales of unregistered stocks in the same publicly traded company. The result is a measure of the value of registration; it is not a measure of liquidity, much less a measure of DLOM. It is not appropriate to increase the calculated DLOM or otherwise reduce the estimate of FMV for lack of registration. Lack of registration is a factor that is subsumed in the time it takes to market an interest in a private company.[6]

Restricted Stock Studies

Restricted stocks are public company stocks subject to limited public trading pursuant to SEC Rule 144. Restricted stock studies attempt to quantify DLOM by comparing the sale price of publicly traded shares to the sale price of otherwise identical marketability-restricted shares of the same company.[7] The average ("mean") marketability discount and related standard deviation (where available) determined by a selection of the published restricted stock studies follows:[8]

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[6] Likewise, brokerage and transactions costs should not be deducted from fair market value appraisals. The result of such deductions would be values that no longer represent the price at which the investments change hands between buyers and sellers – a requirement of fair market value.

[7] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals*, pages 12 and 13

[8] http://www.mercercapital.com/media/Image/ARTICLE_LIBRARY/Tax_Compliance/Rest%20Stock%20Studies%202.gif

PRESENTLY KNOWN RESTRICTED STOCK STUDIES

	Number of Observations	Reported Median	Reported Mean	Reported Standard Deviation
SEC overall average (1966-June 1969)	398	24%	26%	na
Milton Gelman (1968-1970)	89	33%	33%	na
Robert E. Moroney (1969-1972)	146	34%	35%	18%
J. Michael Maher (1969-1973)	34	33%	35%	18%
Robert R. Trout (1968-1972)	60	Na	34%	na
Stryker / Pittock	28	45%	na	na
Willamette Management Associates (1981-1984)	33	31%	na	na
Silber (1981-1988)	69	na	34%	24%
FMV Opinions (Hall / Polacek) (1979-1992)	100+	na	23%	na
FMV Opinions (1991-1992)	na	na	21%	Na
Management Planning, Inc. (1980-1995)	49	29%	28%	14%
Management Planning, Inc. (1980-1995)	20	29%	27%	13%
BVR (Johnson) (1991-1995)	72	na	20%	na
Columbia Financial Advisors (1996-April 1997)	23	14%	21%	na
Columbia Financial Advisors (May 1997-1998)	15	9%	13%	na

In 1997, the SEC reduced the two-year restriction period of Rule 144 to one year.[9] Subsequently, Columbia Financial Advisors, Inc. completed a study that analyzed restricted stock sales from May 1997 through December 1998. This study found a range of discounts from 0% to 30%, and a mean discount of 13%.[10] The conclusion reached from this study is that shorter restriction periods result in lower discounts. In 2008, the SEC further reduced the Rule 144 restriction period to six months.[11] According the IRS, no restricted stock studies have been published that reflect the six-month holding period requirement.[12] Considering the age of the

[9] Securities and Exchange Commission, *Revisions to Rules 144 and 145*, Release No. 33-8869; File No. S7-11-07, at pages 7 and 13, et seq. http://www.sec.gov/rules/final/2007/33-8869.pdf

[10] Mercer Capital, *Restricted Stock Studies Typical Results Do Not Provide "Benchmark."* http://www.mercercapital.com/print/?id=411.

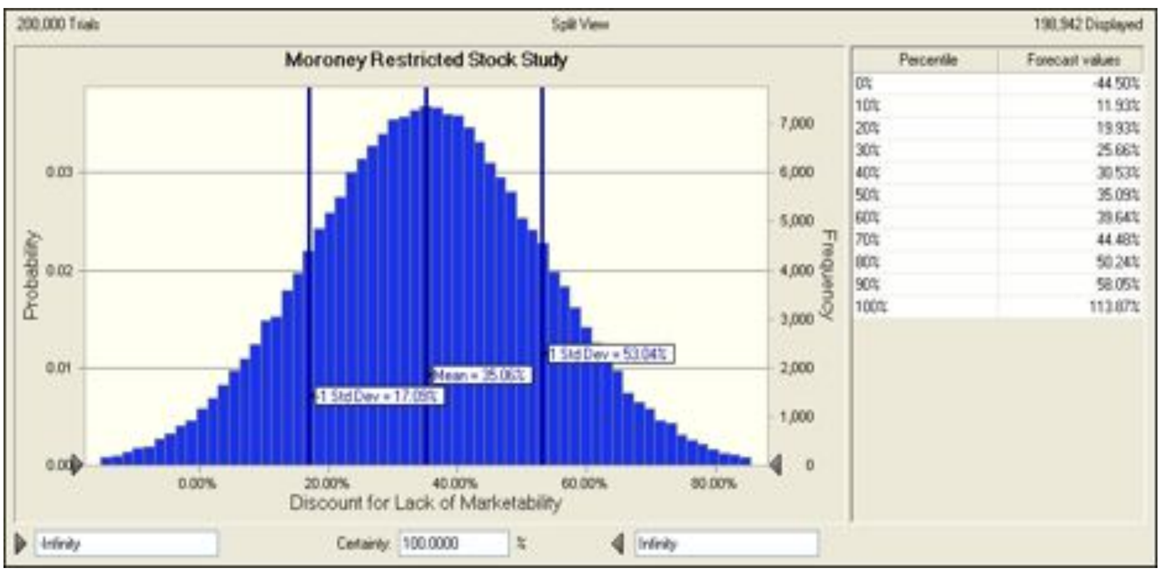
[11] Securities and Exchange Commission, *Revisions to Rules 144 and 145*, Release No. 33-8869; File No. S7-11-07, at pages 13, et seq. http://www.sec.gov/rules/final/2007/33-8869.pdf

[12] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals*, page 17

restricted stock studies, the Rule 144 transitions, and changes in market conditions, concluding that a DLOM derived from the above studies ignores current market data and conditions seems unavoidable.

Appraisers face other serious problems when relying on these studies. Because the sample sizes of the restricted stock studies are small, most involving less than 100 individual data points, the reliability of the summary statistics is subject to considerable data variation.[13] This fact alone calls the reliability of the studies into question. But the studies also report high standard deviations, as shown in the table above, indicating the probability of a very broad range of underlying data points. Relying solely on the averages of these studies is, therefore, likely to lead the appraiser to an erroneous DLOM conclusion:[14]

The graph below was prepared using Oracle *Crystal Ball* to model a 200,000-trial normal statistical distribution based on the reported means and standard deviations of the 146-observation Moroney study. It discloses that the potential range of discounts comprising the 35% mean discount of this study is from *negative* 44.5% to positive 113.9%.



Applying the same normal distribution analysis to the Maher, Silber, and Management Planning studies, we find:

- The potential range of discounts comprising the Maher study average of 35.0% is from *negative* 41.0% to positive 110.6%.

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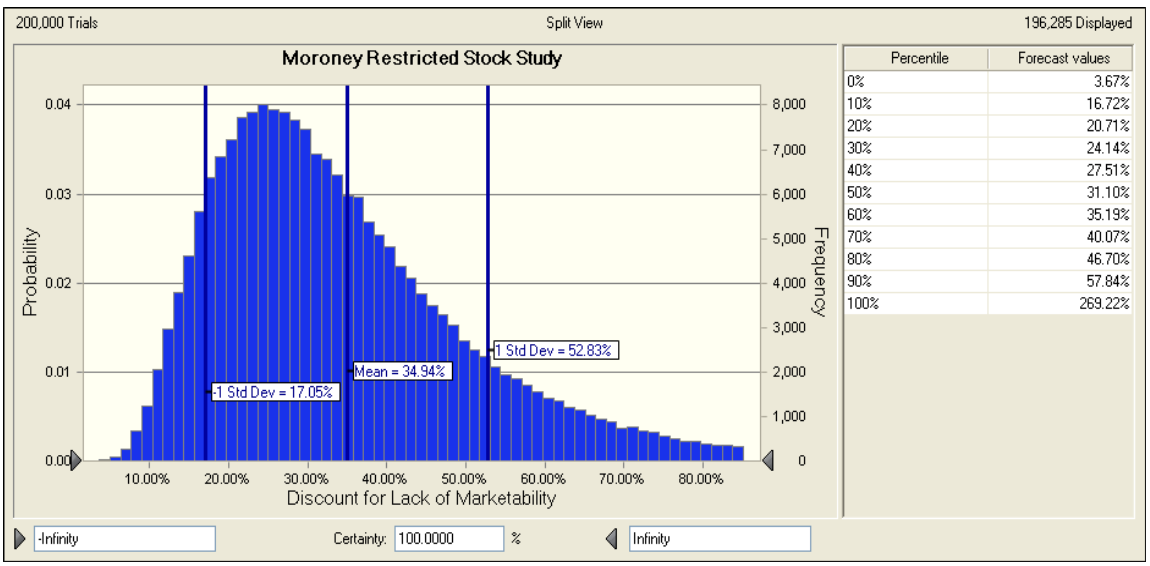
[13] Id. page 15.

[14] Id. page 17.

- The potential range of discounts comprising the Silber study average of 34.0% is from *negative* 75.8% to positive 138.0%.
- The potential range of discounts comprising the 49-observation Management Planning study is from *negative* 32.5% to positive 83.1%.
- The potential range of discounts comprising the 20-observation Management Planning study is from *negative* 29.9% to positive 83.7%.

Common sense tells one that a DLOM cannot be negative. Therefore, normal statistical distribution cannot be the appropriate assumption regarding the distribution of the population of restricted stocks. A log-normal distribution must instead be assumed for the population. Using *Crystal Ball* with the log-normal assumption and 200,000 trials resulted in the graph below. It discloses that the log-normal range of discounts comprising the Moroney study is from 3.7% to 269.2% with a median discount of 31.1%. Approximately 60% of probable outcomes occur below the study mean.



Applying the same log-normal distribution analysis to the Maher, Silber, and Management Planning studies, we find:

- The log-normal range of discounts comprising the Maher study is from 4.0% to 276.6% with a median discount of 31.2%. Approximately 60% of probable outcomes occur below the study mean.
- The log-normal range of discounts comprising the Silber study is from 2.0% to 472.8% with a median discount of 27.8%. More than 60% of probable outcomes occur below the study mean.

- The log-normal range of discounts comprising the Management Planning study is from 2.7% to 233.1% with a median discount of 25.0%. Approximately 60% of probable outcomes occur below the study mean.

Even assuming a log-normal distribution the appraiser is left with two problems. First, what should be done about the fact that some portion of the distribution continues to imply a DLOM greater than 100%? Can that simply be ignored? Is some form of adjustment required? Second, with 60% or more of the predicted outcomes occurring below the reported means of the studies, what is the basis for assuming a DLOM based on a study's mean (or an average of studies' means)? These issues, the inability of the studies to reflect market dynamics (past or present), the inability to associate the studies with a specific valuation date, and the inability to associate the study results to a valuation subject with any specificity, seriously call into question the reliability of basing DLOM conclusions on restricted stock studies.

Pre-IPO Studies

Pre-IPO studies analyze otherwise identical stocks of a company by comparing prices before and as-of the IPO date.[15] As with the restricted stock studies, the valuation utility of the pre-IPO studies is seriously flawed. For example, the "before" dates of these studies use different measurement points ranging from several days to several months prior to the IPO.[16] Determining a "before" date that avoids market bias and changes in the IPO company can be a difficult task.[17] If the "before" date is too close to the IPO date, the price might be affected by the prospects of the company's IPO. If the "before" date is too far from the IPO date, overall market conditions or company specific conditions might have changed significantly. Such circumstances undermine the use of pre-IPO studies to estimate a specific DLOM.

The IRS DLOM Job Aid discusses three pre-IPO studies: the Willamette Management Associates studies; the Robert W. Baird & Company studies; and the Valuation Advisors' Lack of Marketability Discount Study.[18] Each of these studies suffers from deficiencies that undermine their usefulness for estimating the DLOM applicable to a specific business as of a specific date. First, the Willamette and Baird & Company studies were of limited size and are not ongoing. The Willamette studies covered 1,007 transactions over the years 1975 through 1997 (an average of 44 transactions per year), while the Baird & Company studies covered 346 transactions over

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[15] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals,* page 19.

[16] Id.

[17] Id. page 21.

[18] Id. page 19.

various time periods from 1981 through 2000 (an average of 17 transactions per year).[19] While the Valuation Advisors studies are ongoing and larger than the others, covering 9,075 transactions over the years 1985 to present, it represents an average of just 336 pre-IPO transactions per year.[20] Although larger than the restricted stock studies discussed in the previous section, the sample sizes of these pre-IPO studies remain small on an annual basis and subject to considerable data variation.[21] This fact alone calls the reliability of the pre-IPO studies into question.

Second, the Willamette and Baird & Company studies report a broad range of averages, and very high standard deviations relative to their means (reflecting the broad range of underlying data points).[22] The "original" Willamette studies report standard mean discounts that average 39.1% and standard deviations that average 43.2%.[23] The "subsequent" Willamette studies report standard mean discounts that average 46.7% and standard deviations that average 44.8%.[24] And the Baird & Company studies report standard mean discounts that average 46% and standard deviations that average 45%.[25] The graph below was prepared using *Crystal Ball* to model a 200,000-trial normal statistical distribution based on the reported means and standard deviations of the "original" Willamette studies. It discloses that a potential range of discounts comprising the 39.1% mean discount of this study ranges from *negative* 167.6% to positive 235.8%.

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[19] Id.

[20] *See* description of the Valuation Advisors Lack of Marketability Discount Study at http://www.bvmarketdata.com/defaulttextonly.asp?f=Valuation%20Advisors%20Lack%20of%20Marketability%20Discount%20Study%20-%20DLOM%20Database%20(Discount%20for%20Lack%20of%20Marketability)

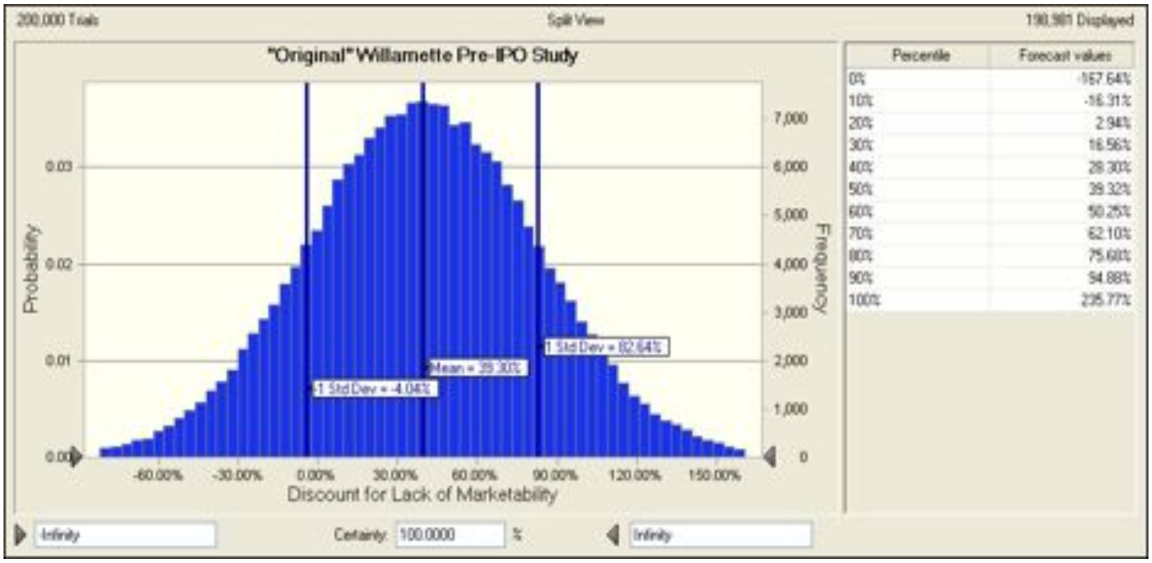
[21] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals,* page 15.

[22] The standard deviation of the Valuation Advisors study is not available on its website.

[23] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals,* page 95.

[24] Id. page 96.

[25] Id. page 97.



Applying the same normal distribution analysis to the "subsequent" Willamette studies and the Baird & Company studies, we find:

- A 206-observation subset of the aforementioned Baird & Company studies reports average mean discounts of 44% and average standard deviations of 21%.[26] The potential range of discounts comprising this study is from *negative* 59.8% to positive 150.6%.

As with the restricted stock studies, common sense tells one that a DLOM cannot be negative. Therefore, normal statistical distribution cannot be the appropriate assumption regarding the distribution of discounts within the populations, and a log-normal distribution must be assumed instead. Using *Crystal Ball* with the log-normal assumption and 200,000 trials resulted in the graph below. It discloses that the log-normal range of discounts comprising the "original" Willamette study is from 0.5% to 1151.2% with a median discount of 26.3%. Almost 70% of probable outcomes occur below the 39.1% mean discount of the study.

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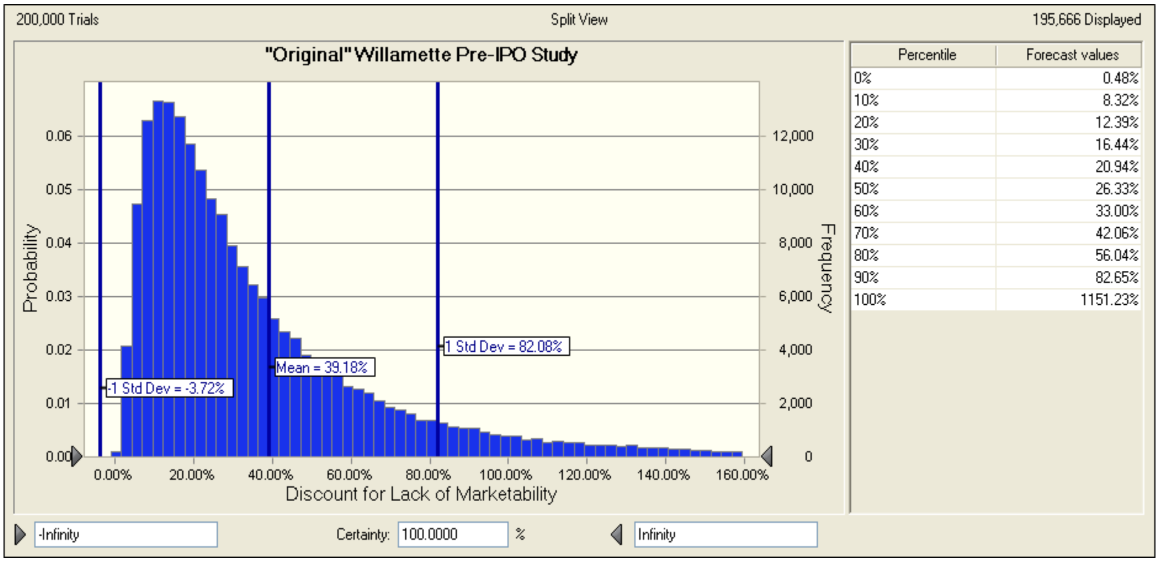
[26] Z. Christopher Mercer, Quantifying Marketability Discounts (2001), page 80.



- The potential range of discounts comprising the "subsequent" Willamette studies is from 1.3% to 1,192.9% with a median discount of 33.8%. Over 60% of probable outcomes occur below the mean discount of the study.
- The potential range of discounts comprising the Baird & Company studies is from 5.7% to 327.3% with a median discount of 42.7%. Approximately 60% of probable outcomes occur below the mean discount of the study.

These statistical problems of the pre-IPO studies and the inability to (a) align with past and present market dynamics; (b) a specific valuation date; and (c) a specific valuation subject, seriously call into question the reliability of basing DLOM conclusions on pre-IPO studies.

Third, the volume of IPO transactions underlying the pre-IPO studies is shallow and erratic as shown in the graph below:[27]

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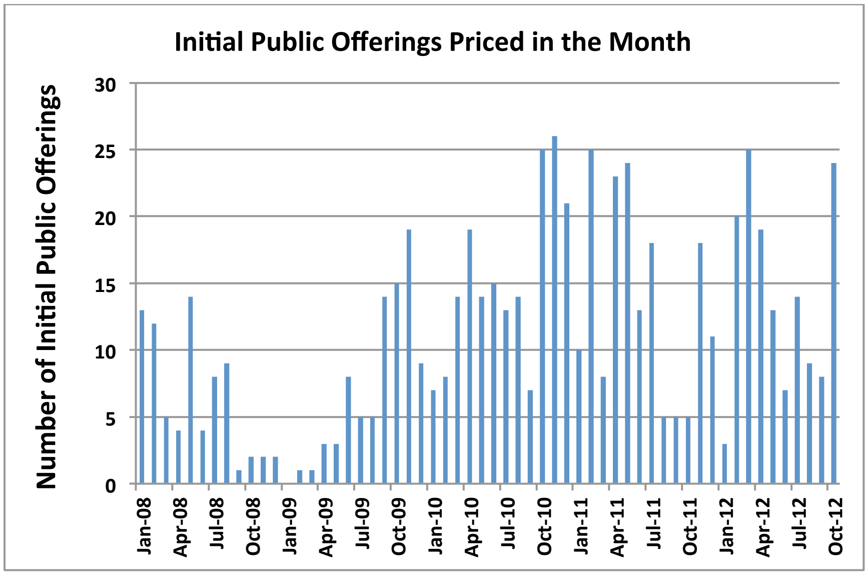
[27] http://www.nasdaq.com/markets/ipos/activity.aspx?tab=filings



In the last approximately five years the peak volume of offerings was 26 (November 2010) and in January 2009 there were no IPOs at all. From September 2008 through March 2009 the average number of IPOs priced was less than 1.3 per month. It is difficult to understand a rationale for estimating DLOM for a specific privately held company at a specific point in time based on such limited data.

Fourth, the Tax Court has found DLOM based on the pre-IPO approach to be unreliable. In *McCord v. Commissioner* the court concluded that the pre-IPO studies may reflect more than just the availability of a ready market. Other criticisms were that the Baird & Company study is biased because it does not sufficiently take into account the highest sales prices in pre-IPO transactions and the Willamette studies provide insufficient disclosure to be useful.[28]

PROBLEMS WITH SOME EXISTING ANALYTICAL METHODS TO MEASURE DLOM

It has been suggested that the Black-Sholes Option Pricing Model ("BSOPM") represents a solution to the DLOM conundrum. It does not. BSCPM is not equivalent to DLOM on a theoretical basis. BSOPM is designed to measure European put and call options. European put options represent the right, but not the obligation, to sell stock for a specified price at a specified point in time. European call options represent the right, but not the obligation, to buy stock for a specified price at a specified point in time. DLOM is not the equivalent of either. Instead, DLOM represents the risk of being unable to sell at any price for a specified period of time.

"At the money" put options have also been suggested as a means of estimating DLOM. Such options represent the right, but not the obligation, to sell stock at the current price at a

[28] *McCord v. Commissioner,* 120 T.C. 358 (2003)

specified future point in time. Such options do not measure the risk of illiquidity, because the investor is not denied the opportunity to sell for a price that is higher than the put price.

THE LONGSTAFF APPROACH FOR COMPUTING DLOM

The critical value difference between publicly traded and privately held companies is that publicly traded investments offer liquidity. All other components of business value are shared: earnings and cash flow, growth, industry risk, size risk, and market risk. However, it is not the value of liquidity per se that DLOM seeks to capture. Instead, it is the ***risk*** associated with ***illiquidity.***

But first, what is liquidity? It is the ability to sell quickly when the investor decides to sell. Liquidity allows investors to sell investments quickly to lock in gains or to avoid losses. DLOM, being the result of ***illiquidity,*** represents the economic risk associated with failing to realize gains or failing to avoid losses on an investment during the period the investor is trying to sell it. This is not necessarily a zero sum game. The value of liquidity (such as observed by Bajaj, et al.) does not translate into the economic risks of faced by investors in private companies. This is because the Bajaj approach does not account for the even longer marketing periods likely to be incurred by investors in private companies compared to investors in unregistered stocks of otherwise publicly traded companies.

Logically, DLOM can be reduced to price risk faced by an investor during a particular marketing period. In the market for publicly traded stocks, risk reflects the volatility of stock prices. Conversely, investments with no price volatility or that are immediately marketable have no DLOM. Investments with no price volatility can be arbitraged to negate the period of restricted marketing, while volatile investments that are immediately marketable can be sold at the current price to avoid future volatility.

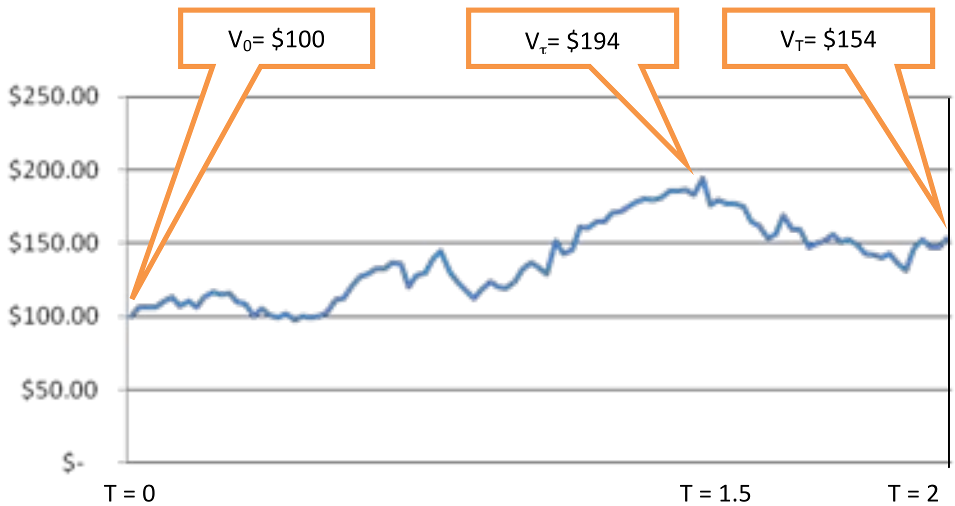
In 1995, UCLA professor Francis A. Longstaff published an article in The Journal of Finance[29] that presented a simple analytical upper bound on the value of marketability using "lookback" option pricing theory. Longstaff's analysis demonstrated that discounts for lack of marketability ("DLOM") can be large even when the illiquidity period is very short. Importantly, the results of Longstaff's formula provide insight into the relationship of DLOM and the length of time of a marketability restriction. Longstaff described the "intuition" behind the results of his formula as follows –

> [Consider] a hypothetical investor with perfect market timing ability who is restricted from selling a security for *T* periods. If the marketability restriction were to be relaxed, the investor could then sell when the price of the security reached its maximum. Thus, if the marketability restriction were relaxed, the incremental

[29] The Journal of Finance, Volume I, No. 5, December 1995

cash flow to the investor would essentially be the same as if he swapped the time-*T* value of the security for the maximum price attained by the security. The present value of this lookback or liquidity swap represents the value of marketability for this hypothetical investor, and provides an upper bound for any actual investor with imperfect market timing ability.

Figure 3



For this sample path:

- With restriction, present value of T = 2 at T = 0 is 154*exp(-2*.05) = $139
- Without restriction, could have 194*exp(-1.5*.05) = $180 present value
- Cost of restriction is the difference in present values = $180 - $139 = $41
- DLOM percentage = present value difference divided by investment = 41/100 = 41%

Figure 3 is a graphic presentation of Longstaff's description, in which an investor receives a share of stock worth $100 at time zero, but which he cannot sell for *T* = 2 years when the stock is worth $154 (present value at *T* = 0 discounted at a risk free rate of 5% = $139). If at its peak value the stock were worth $194 (present value at *T* = 0 discounted at a risk free rate of 5% = $180), then the present value cost of the restriction to the investor at *T* = 0 would be $41, or 41% of his $100 investment. The mathematical formula of this scenario is –

$$Discount = V\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right)+V\sqrt{\frac{\sigma^2 T}{2\pi}}\exp\left(-\frac{\sigma^2 T}{8}\right)-V$$

where:

$V = current\ value\ of\ the\ investment$

$\sigma = volatility$

$T = marketability\ restriction\ period$

$N = standard\ normal\ cumulative\ distribution\ function$

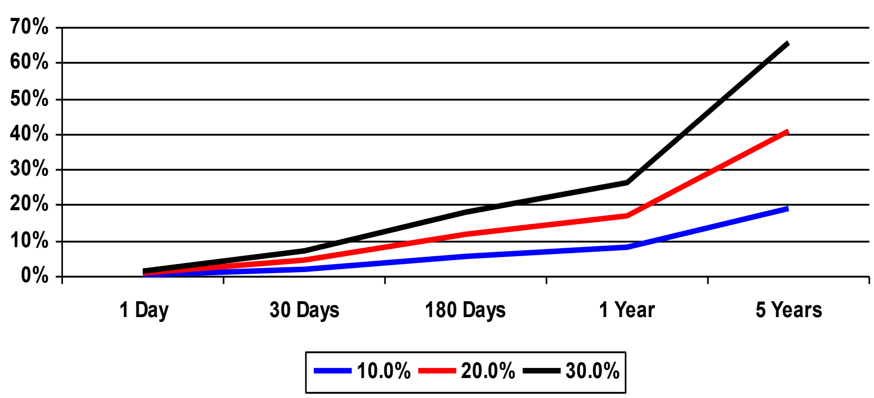
The following table presents the results of the formula at various combinations of volatility and length of time of restrictions on marketability.

	Volatility		
	10%	20%	30%
1 Day	0.4%	0.8%	1.3%
30 Days	2.3%	4.7%	7.1%
180 Days	5.8%	11.8%	18.1%
1 Year	8.2%	17.0%	26.3%
5 Years	19.1%	41.0%	65.8%

Figure 4 presents the results graphically:

Figure 4

COMPARISON OF RESULTS OF LONGSTAFF FORMULA WITH VARIOUS VOLATILITIES AND RESTRICTION PERIODS



THE MARKETING PERIOD OF PRIVATE SALE TRANSACTIONS

The business valuation concept of marketability deals with the liquidity of the ownership interest.[30] How quickly and certainly an owner can convert an investment to cash represent two very different variables. The "quickly" variable represents the period of time it will take the seller to liquidate an investment. This period of time can vary greatly depending on the standard of value in play. For example, liquidation sales can occur quickly and generally reflect lower prices, while orderly sales usually take longer to explore the marketplace of reasonable buyers and generally reflect greater than liquidation prices. In every instance, however, the "quickly" variable commences with a decision by the seller to initiate the sales process.

The marketing period of a privately held business is seldom less than a few months, and can be much longer, as the following events occur:

- Drafting selling documents
- Developing a marketing strategy
- Implementing the marketing strategy
- Screening buyers
- Conducting site visits
- Assisting buyers in their analysis of the company and the interest being sold
- Drafting letters of intent
- Negotiating with the serious buyers
- Assisting buyers with due diligence
- Drafting the contract of sale
- Participating in arranging financing
- Actually closing the deal

The "certainty" variable represents the probability that the seller will realize the estimated sale price (value) of the investment. Therefore, the "certainty" variable represents the price volatility of the investment during the period of time that it is being offered for sale. If market prices for similar investments fall dramatically while the marketplace is being explored, then the seller will have lost the opportunity to lock in the higher price that existed at the time the sell decision was made. Conversely, if the sale price is fixed for some reason (e.g., a listing agreement) and market prices for similar investments rise dramatically during the marketing period, the seller will have lost the opportunity to realize the increased value.

The "quickly" and "certainty" variables work together when determining the value of an investment. Relative to immediately marketable investments, the value of illiquid investments (regardless of the level of value) must be discounted to reflect the uncertainty of the time and

[30] Shannon P. Pratt and Alina V. Niculita, *Valuing a Business, 5th Edition: The Analysis and Appraisal of Closely Held Companies.* (McGraw-Hill, 2007), page 417.

price of sale. This uncertainty is reflected in business valuations by what is commonly known as the “discount for lack of marketability” (“DLOM”).

Logically, the economic costs of time and price uncertainty can be reduced to the price risk faced by an investor during the particular period of time that an illiquid investment is being offered for sale. In the market for publicly traded stocks, the volatility of stock prices represents risk. Investments with no price volatility have no DLOM, because they can be arbitraged to negate the risk of a period of restricted marketing. Conversely, volatile investments that are immediately marketable can be sold at the current price to avoid the risk of future volatility. The illiquidity experienced by the seller of a non-public business interest during the marketing period therefore represents an economic cost reflective of the risk associated with the inability to realize gains and to avoid losses during the period of illiquidity.[31] The longer that time period, the more the value of the business is exposed to adverse events in the marketplace and adverse changes in the operations of the business, and the greater the DLOM that is required to equate the investment to an immediately liquid counterpart. The economic cost associated with a period of illiquidity can be estimated using the look-back formula developed by Francis A. Longstaff, Ph.D. in 2002,[32] which relies on estimates of price volatility (i.e., the *certainty* variable) and marketing time (i.e., the *quickly* variable).

Price Volatility Considerations

Price volatility is easily determined if the appraiser can identify at least one appropriate publicly traded company to use as a benchmark.[33] This is obviously a matter of professional judgment. At VFC, we use the same companies for price volatility determination that we use to apply the publicly traded guideline valuation method. We calculate the annualized average stock price volatility and standard deviation for each of the guideline companies for an historic period of time equal that we consider to be predictive of the period of time that we believe it will take to market the interest being valued.[34] We then average the calculated means and standard deviations volatilities using a simple average or harmonic average as called for by the valuation

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[31] Id.

[32] Francis A. Longstaff, “How Much Can Marketability Affect Security Values?”, The Journal of Finance, Volume I, No. 5, December 1995.

[33] The use of guideline companies to estimate the subject company’s stock price volatility is consistent with the requirements of SFAS 123(R) at paragraph 23 and A22.

[34] Subject to possible adjustment described in SFAS 123(R), using the historical volatility of stock over the most recent time period corresponding in length to the expected period of restriction is consistent with the requirements of the pronouncement. See paragraph A21.

purpose.[35] We generally favor simple averages when applying guideline factors in business valuation because the goal is to determine the fair market value of a particular investment. Harmonic averages may be useful, however, if the goal is to create a portfolio of investments that mirrors a particular market. Regardless of the averaging convention selected by the appraiser, basing price volatility estimates on guideline company stock price fluctuations eliminates the "upper bound" objections that some critics have of the Longstaff formula by yielding a discount reflective of average price volatility instead of peak price volatility.

As with guideline company selection, the methodology for predicting future price volatility requires professional judgment. Appraisers may reasonably employ other ways of predicting price volatility than described above.

Marketing Period Considerations

To evaluate the period of time that it takes to sell privately held businesses, we obtained a database of 8,184 private company sale transactions from BV Resources.[36] The population of transactions occurred from February 1992 through the end of 2010, and reported an associated Standard Industrial Classification ("SIC") code; sale initiation date; sale closing date; market value of invested capital ("MVIC"); and asking price. The average time that elapsed from the initial offering date to the closing date of these transactions is 200 days. The standard deviation of the reported time periods is 97.7%, or 195 days. Graph 1 shows the distribution of the amount of time it took to consummate the sale transactions in the database. Since the marketing time period cannot be less than zero days, the distribution of the database obviously skews to the right. The data is split into 30-day increments for presentation and analytical purposes.

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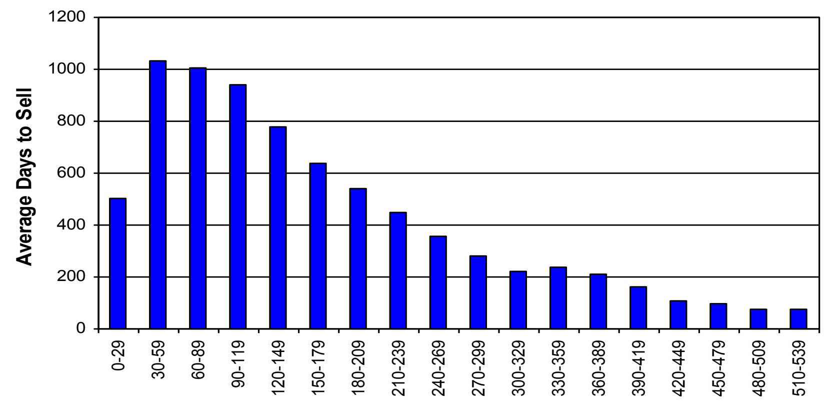
[35] On occasions, we will average the volatilities of the guideline companies using a weighted average that reflects the companies' relative participation in the industry of the subject company.

[36] Pratt's Stats® is the BV Resources database where the transactions were obtained. We did not investigate the accuracy with which transactions are reported in the database.

Graph 1
Average Marketing Period by Year of Sale Initiation
8,184 Private Sales Transactions



Graph 1 shows that the population of sale transactions follows a logarithmic distribution. The peak of the graph is 1,032 sale transactions that occurred from 30 to 59 days to sell, which is 12.6% of the database.[37] The database analysis indicates that one standard deviation to the right of the mean encompasses marketing periods of up to 395 days, which is 88% of the database population.

Graph 1 was then compared to a distribution created using the population's mean and standard deviation and Oracle's *Crystal Ball* software. Graph 2 shows the *Crystal Ball* output using a log-normal distribution[38]:

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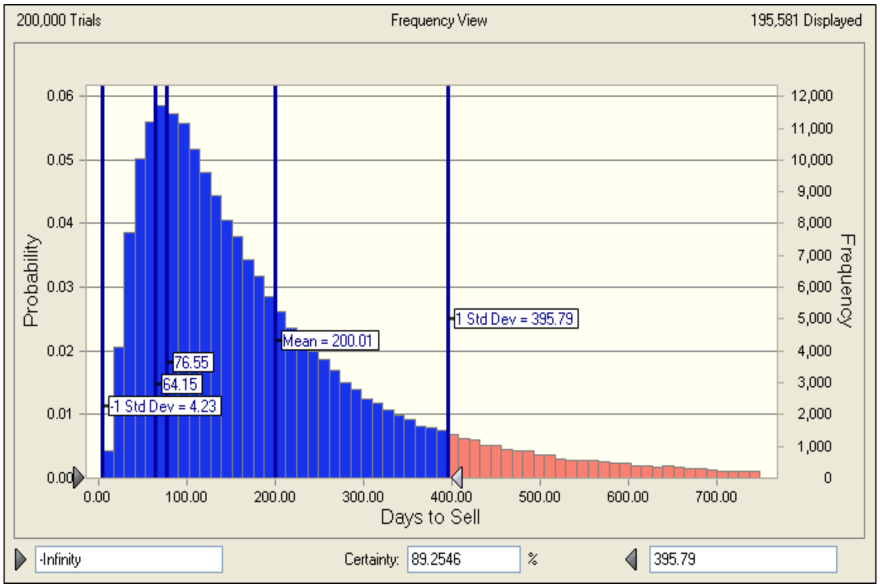
[37] When the sales are presented on single-day time periods, spikes in the frequency of sales transactions occur about 30 days apart. This could be the result of faulty information supplied by brokers, or a tendency of sales to occur at the end of listing agreements. We used 30-day periods to eliminate the distortion of the spikes.

[38] A log-normal distribution is positively skewed, with most values near the lower limit and is based on natural logarithms.

Graph 2



Graph 2 shows that the peak frequency of sale events is 5.9%, which occurs from the range of approximately 64.2 to 76.6 days. But Graph 2 is based on 12-day, not 30-day, intervals. Adjusted ratably to a 12-day interval, the peak probability of Graph 1 is 5.0%. And as with the actual database, the *Crystal Ball* analysis indicates that one standard deviation to the right of the mean encompasses marketing periods of up to 396 days, representing 89% of the database population.[39] Therefore, the database population follows the log-normal distribution of *Crystal Ball*, which we use for the remainder of this article.

Marketing Periods Based on Industry

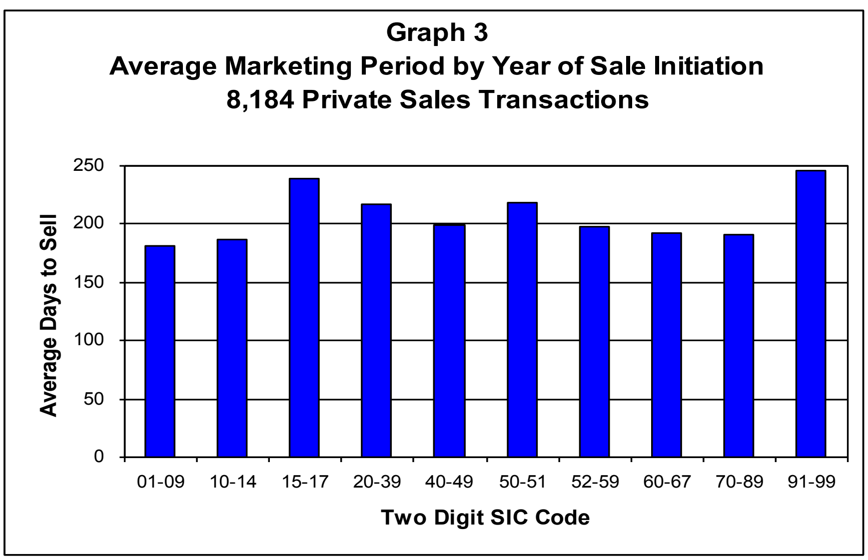
Now let's see what happens when we dig deeper. We separated the sale transactions into the ten two-digit SIC code divisions corresponding to the broad industry groupings shown in Table 1 and Graph 3. The description, number of private sale transactions, and average days to sell is listed for each industry group. The standard deviations of these industries range from 143 days to 257 days.

[39] The 89.2546 "certainty" shown in Graph 2 is not a probability certainty. Instead it is an absolute measure of the percentage of the population represented by one standard deviation to the right of the mean. See Crystal Ball User Manual at p.100.

Table 1

SIC Code	SIC Group	Number of Sale Transactions	Average Selling Time in Days
01-09	Agriculture, forestry, and fishing	269	182
10-14	Mining	7	187
15-17	Construction	379	239
20-39	Manufacturing	927	216
40-49	Transportation, communications, electric, gas, and sanitary services	248	199
50-51	Wholesale trade	510	219
52-59	Retail trade	2,949	197
60-67	Finance, insurance, and real estate	152	193
70-89	Services	2,741	191
91-99	Public administration	2	246
	All industries	8,184	200

Graph 3 depicts the variation in the calendar month averages from Table 1:



The three industry groups of construction, wholesale trade, and manufacturing had the longest marketing periods, with averages of 239, 219, and 216, respectively.[40] Businesses

[40] We are ignoring the public administration industry group since it represents the sale of just two businesses.

reported in the agriculture, forestry, and fishing industries sold quickly in an average of 182 days. On average, businesses in the transportation, communications, electric, gas, and sanitary services industry group sold within 199 days; businesses in the retail industry group sold within 197 days; businesses in the financial, insurance, and real estate industry group sold within 193 days; businesses in the services industry group sold within 191 days; and businesses in the mining industry group sold within 187 days. The standard deviations of the marketing periods of the industries also varied greatly.

The 57-day spread between the 239-day average selling period of construction businesses and the 182-day average selling period of agriculture, forestry, and fishing businesses demonstrates that industry makes a material difference in how long it is likely to take to close the sale of a business. Adding widely varying standard deviations of marketing periods to the various mean marketing periods of different industries highlights the very different marketing period risks faced by owners of businesses engaged in different industries.

Marketing Periods Based on Sale Year

The next factor explored is the effect on the marketing period of the calendar year in which the businesses were listed for sale. The BV Resources database reports sale transactions commencing in 1991 and extending through 2010. The years 1991 to 1995 were not used in the calendar year analysis since there were very few listings from these years. Excluding 1991 through 1995 reduced the database population from 8,184 to 8,103. Calendar years 2009 and 2010 were also not used in the calendar year analysis because the closing dates of these listings are not yet known. Excluding 2009 and 2010 reduced the database population from 8,103 to 6,940.

Table 2 shows the average marketing period and number of transactions by year for sales listed from 1996 through 2008:

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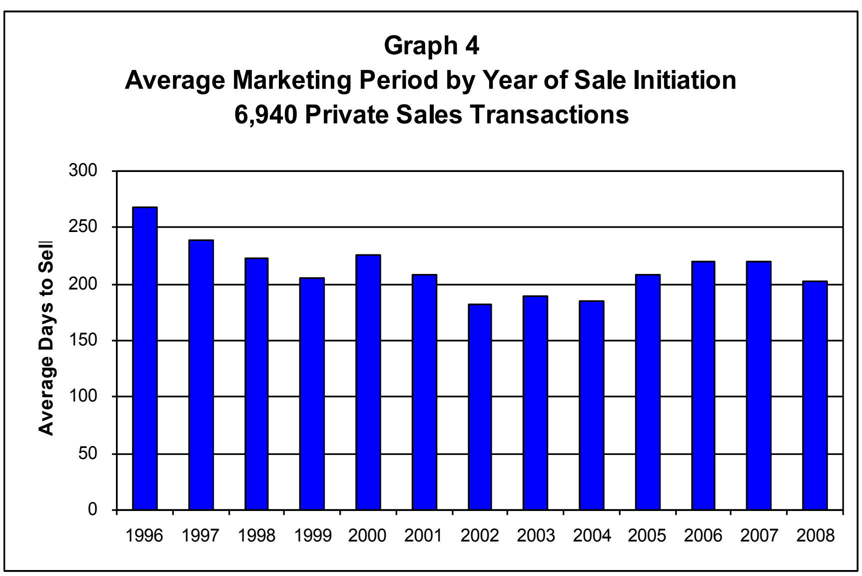
Table 2

If Listed In	2007 Study Average Selling Time in Days	2008 Study Average Selling Time in Days	Number of Transactions
1996	265	267	71
1997	240	239	133
1998	211	223	250
1999	204	206	270
2000	218	226	372
2001	200	209	440
2002	172	182	519
2003	178	189	521
2004	175	185	737
2005	189	208	748
2006	195	220	819
2007	166	220	1,112
2008		202	948
Average	**201**	**214**	

Graph 4 shows the declining trend of average selling periods over time. The average number of days it took to sell the privately held businesses in the study decreased from 267 days in 1996 to 182 days in 2002, before increasing to 220 days in 2007 and falling to 202 days in 2008.



Our earlier study postulated that GDP, inflation, money supply, and demographics could explain the declining trend of the marketing periods. Correlation analysis of selling time against these factors yielded low R-squares, suggesting that annual fluctuations in inflation, real GDP, nominal GDP, money supply, and demographics provide little explanation of the declining trend of private business marketing periods. The earlier study was confirmed by the present study.

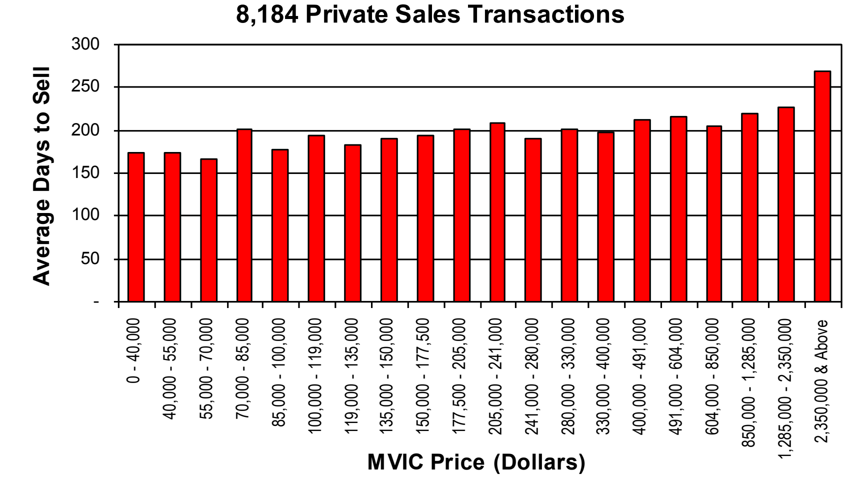
During the period of the analyzed database, there was a recession from March to November in 2001. This possibly explains the longer selling times for those sales that were listed in 2000 and closed in 2001, but the explanation is seemingly contradicted by the decline in the average number of days to sell businesses listed in 2001. Despite the recession, the average business sold faster during 2001 than in 2000. A major recession also started in December 2007. This possibly explains the longer selling times for those sales that were listed in 2006 and 2007, which were both 5.8% longer than sales listed in 2005. One might expect these listings to take longer to close if they were initiated but not completed by the start of the recession.

Marketing Periods Based on Price

The BV Resources database of transactions also provided the MVIC and asking price of each transaction. MVIC is the market value of invested capital comprised of all stock classes and interest-bearing debt. The MVIC and asking price factors were used to separately analyze the database.

The range of MVIC was $1,000 to $314,000,000. The mean and median MVIC of the population was $783,067 and $205,000, respectively. The sale transactions were split into 20 groups based on MVIC. The MVIC range of the group intervals becomes larger as MVIC increases. Each size group contains 409 sale transactions except the largest group, which contains 413. Graph 5 shows the average days to sell for each MVIC group.

Graph 5
Average Marketing Period by MVIC Price
8,184 Private Sales Transactions

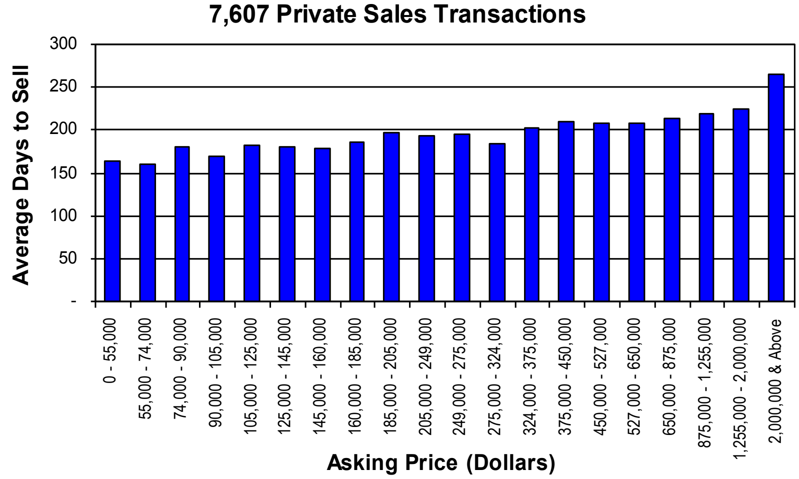


Generally, the average days to sell increases with the rise in MVIC. When the MVIC is under $40,000, the average days to sell is 173 days. The length of marketing periods gradually increases until the MVIC price is greater than $2,350,000, when the average days to sell is 269 days.

Exponential regression of the average marketing periods of the MVIC groups yielded a fairly strong R-square of 73%. The regression formula shows that the average days to sell increases by 1.6% as MVIC progresses from group to group. The trend line predicts 171 days to complete a sale transaction when the MVIC is below $40,000. When the MVIC is above $2,350,000, the trend predicts at 231 days to sell, but the actual marketing time of this group is much higher as the graph shows.

Some of the transactions did not report asking prices. Those sale transactions were removed from this analysis, which reduced the database population for the asking price analysis from 8,184 to 7,607. The mean of the 7,607 sale transactions is 196 days to sell. The range of asking prices of the resulting population of sale transactions was from $3,456 to $70,000,000. The mean and median asking prices of the population were $608,018 and $249,000, respectively. Each size group contains 380 sale transactions except the largest group, which contains 387. Graph 6 shows the average days to sell for each asking price group.

Graph 6
Average Marketing Period by Asking Price
7,607 Private Sales Transactions



The fluctuations in the asking price graph are generally similar to those of the MVIC graph. When the asking price is under $55,000, the average days to sell is 164 days. The length of the marketing period gradually increases until the average days to sell is 265 days when the asking price is greater than $2,000,000.

Exponential regression of the average asking price of each group resulted in a strong 86% R-square. The regression formula shows that the average days to sell increases by 1.9%, as asking price progresses from group to group. The regression predicts that it takes 163 days to complete a sale transaction when the asking price is below $55,000. When the asking price is above $2,000,000, the regression predicts that it takes 232 days to close a sale. However, note that the 265-day average marketing period for businesses priced higher than $2 million is significantly above the trend number.

As mentioned, the asking price regression yields a stronger R-square of 86%[41] while the MVIC regression yields a weaker R-square of 73%. The higher R-square value associated with asking price may be due to reporting inaccuracies that we did not investigate. But it may also reflect that asking price is determinative in drawing potential buyers to the sale opportunity. Assuming no database adjustments are warranted, the asking price is the better statistical predictor.

Marketing Periods Based on Seasonality

We also considered whether the time of year a sale transaction is initiated makes a difference in the length of marketing periods. To analyze this factor, the sale transactions were grouped based on the month the company was listed to sell. Table 3 reports the mean number of days to sell that elapsed from the listing date based on a distribution of the sale transactions according to the calendar month the businesses were listed for sale:

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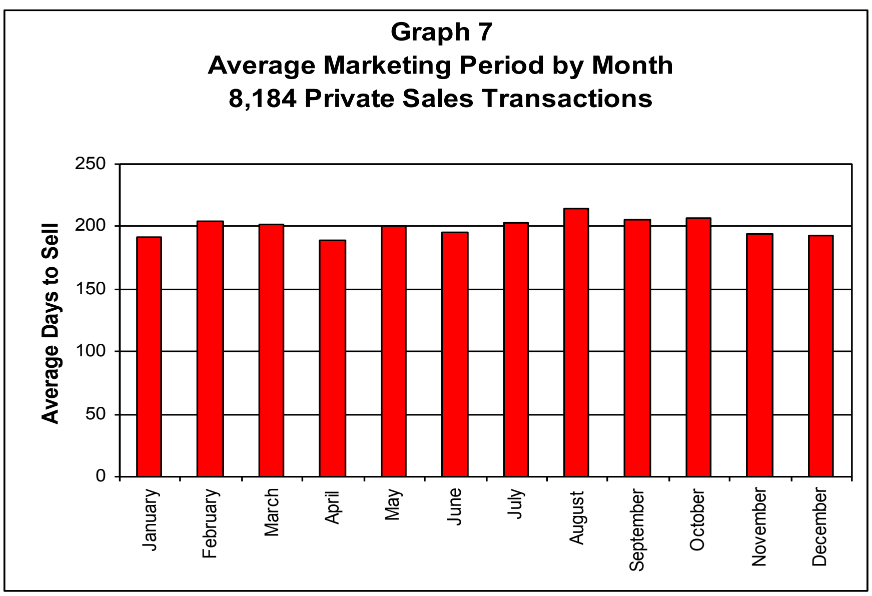
[41] A linear regression resulted in an R-square value of 83%. The slope was 3.7, meaning for each increase from one asking price group to another, the average days to sell increases by 3.7 days.

Table 3

If Listed In	Number of Sale Transactions	Average Days to Sell
January	774	192
February	682	204
March	740	201
April	697	190
May	686	200
June	697	195
July	694	203
August	678	214
September	642	206
October	689	207
November	611	194
December	594	193
Average		**200**

Graph 7 depicts the variation in the calendar month averages from Table 3:



On average, sale transactions originally listed in August took the longest time to sell, with a mean of 214 days. March listings had the highest volatility of time to sell. Sale transactions originally listed in October also were lengthy, averaging 207 days to sell. The months with the shortest marketing periods were April, January, December, and November averaging 190, 192, 193, and 194 days, respectively. Possible explanations for these phenomena are proximity to yearend numbers for November, December, and January listings, and proximity to completion of tax filings for April listings. Such proximity tends to offer buyers enhanced transparency through timelier financial reporting.

REBUTTING CRITICS OF THE LONGSTAFF DLOM METHODOLOGY

In 1995, when Francis A. Longstaff, Ph.D. presented his idea that the formula for calculating the value of a look back option with and without a liquidity restriction assumption could be used to estimate the discount for lack of marketability ("DLOM") of a financial instrument, he described his approach as quantifying the cost of illiquidity for an investor with otherwise perfect market timing ability. But Dr. Longstaff also recognized that the value of marketability, and therefore the cost of illiquidity, is less for investors with less than perfect market timing ability. Consequently, Dr. Longstaff described his approach as the "upper bound" of DLOM calculations. Since 1995, criticisms of what is now known as the Longstaff methodology have focused on three perceived defects: (1) no investor has perfect knowledge; (2) a DLOM based on an upper bound is excessive; and (3) the look back option formula "breaks down" with long marketing periods and high price volatilities. Each of these criticisms is wrong for the reasons described below.

The "Perfect Knowledge" Criticism

The "perfect knowledge" criticism is based on a defective definition of market timing in a valuation context. The context considered by Dr. Longstaff was one of an investor looking back in time to observe precisely when an investment could have been sold at its maximum value. Dr. Longstaff implicitly assumed that the maximum price could have been reached at any point during the look back period. But in a valuation context this reasonable assumption is not appropriate. Instead, the maximum price occurs on the valuation date and is the marketable value of the valuation subject. Appraisers determine this value in the ordinary course of their work.

Standing on the vantage point of the valuation date and applying look back option pricing to calculate DLOM in a business valuation inherently assumes that the maximum price that the investor could have realized for the investment is the marketable equivalent price as of that date. The value of the investment beyond the valuation date is necessarily less. This is because the time value of money diminishes the present value of the marketable equivalent price over the course of the marketing period; the foreseeable favorable events affecting the valuation subject have been factored into the analysis; and investors are averse to the risks of price volatility. Thus, if the appraiser properly determined the marketable equivalent price as of the valuation date, then that price is the "maximum value" postulated by Dr. Longstaff.

The "Upper Bound" Criticism

Dr. Longstaff described the framework in which an upper bound on the value of marketability is derived as one lacking the assumptions about informational asymmetries, investor preferences, and other variable that would be required for a general equilibrium model. Dr. Longstaff recognized that the cost of illiquidity is less for an investor with imperfect market timing than it is for an investor possessing perfect market timing. These considerations are the basis of the "upper bound" limitation of the Longstaff methodology.

It is irrefutable that the cost of illiquidity must be less for the average investor with imperfect market timing than it is for an investor possessing perfect market timing. But the "upper bound" criticism resulting from this situation is nonetheless defective in the valuation context because it is easily circumvented by using volatility estimates that represent average, not peak, volatility expectations. For example, the appraiser's volatility estimate may be based on some average or regression of historical price volatility derived from an index or from one or more publicly traded guideline companies. Using average volatility estimates in the look back option formula necessarily results in a value that is less than the "upper bound" value. Indeed, a value calculated using average expected volatility necessarily suggests a result that is achievable by the average imperfect investor. The resulting value determined in this manner appropriately falls short of a value based on perfect market timing while providing an important informational asymmetry lacking in Dr. Longstaff's more simplified framework.

Enhanced estimates of DLOMs applicable to average investors can also be crafted by determining the average marketing period required to sell privately held businesses, and the standard deviation of distribution around the mean.[42] Using probability weighted marketing periods therefore provides a second important informational asymmetry lacking in Dr. Longstaff's framework.

Additional framework enhancements include determining the rate of incline or decline in future volatility, and weighting future volatility estimates according to the probability of sale associated with the time period in which the estimates are expected to occur. Accordingly, the "upper bound" criticism has no significance in a proper application of the Longstaff methodology.

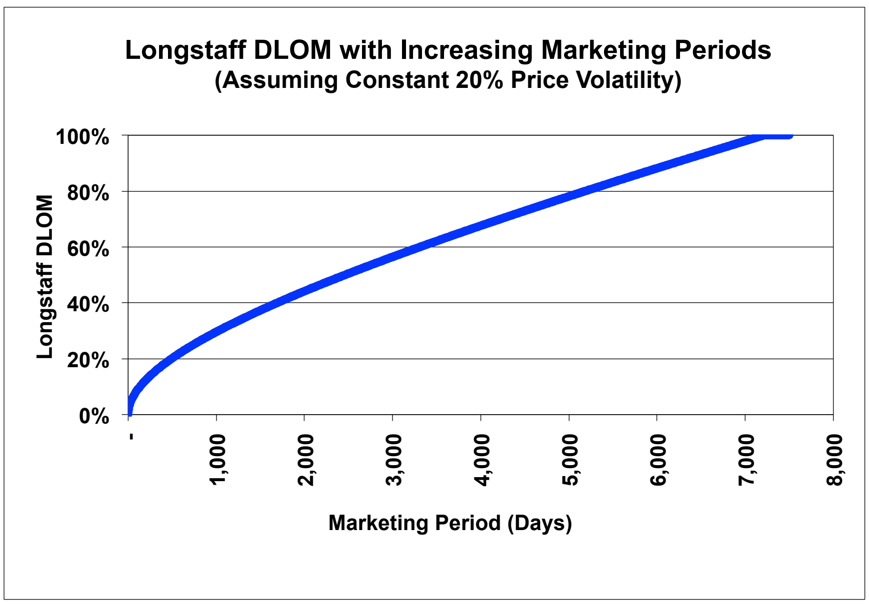
The "Formula Breaks Down" Criticism

The IRS publication "Discount for Lack of Marketability – Job Aid for IRS Valuation Professionals" makes the statement that volatilities in excess of 30% are not "realistic" for estimating DLOM using look back option pricing models. In support of this contention, the

[42] e.g., Vianello, "The Marketing Period of Private Sale Transactions: Updated for Sales through 2010," *Business Valuation Update*, Vol. 17, No. 11, November 2011.

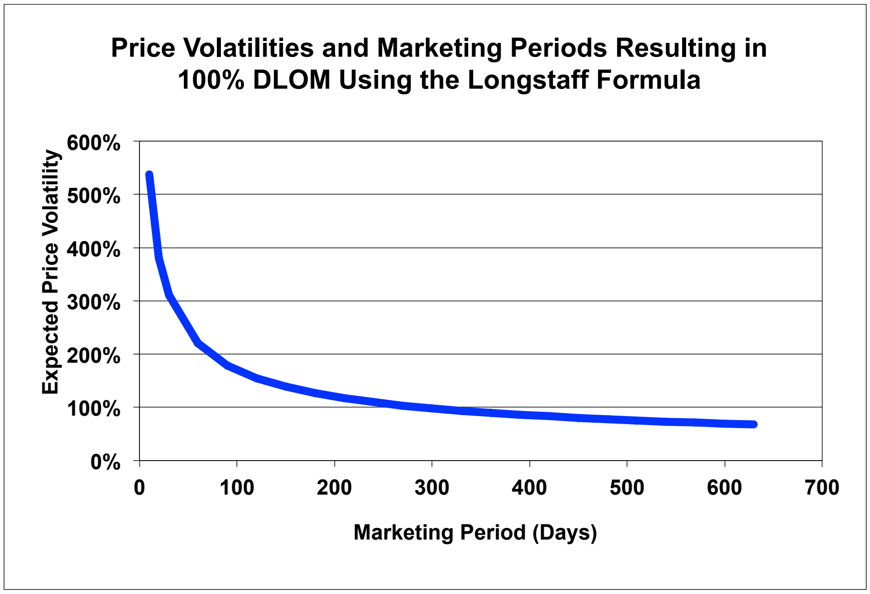
publication provides a table reporting marketability discounts in excess of 100% resulting from using combinations of variables of at least 50% volatility with a 5-year marketing period and 70% volatility with a 2-year marketing period. When that occurs, Longstaff DLOM values should simply be capped at 100%. After all, the criticism is not that the formula incorrectly calculates DLOMs below the 100% limit; merely that DLOM cannot exceed 100%.

The following graph shows the Longstaff DLOM values, capped at 100%, that result from a 20% price volatility assumption and a broad range of marketing periods. The 20% price volatility assumption approximates the historical mean of the VIX from January 2, 1990, to June 30, 2011. Note that it takes about 6,970 days – over 19 years – for the discount to reach 100% with a 20% price volatility assumption. Considering that the typical business sells in about 200 days, a criticism based on a 19-year marketing period is clearly unreasonable.[43]



Of course, as the expected price volatility increases, a shorter time is required to reach 100%. Conversely, as the expected price volatility decreases, a longer time is required to reach 100%. The graph below shows the line demarking varying combinations of price volatility and marketing periods above which Longstaff DLOM values exceed 100%. Considering that the peak volatility of the VIX was about 80% (occurring on November 20, 2008) and that the average period of time in which a private business sells is about 200 days, it is unlikely that typical appraisers will define look back option variables that result in Longstaff DLOM values that exceed 100%.

[43] The VIX peaked at 80.86% on November 20, 2008. With that assumption, the Longstaff formula requires a 450-day lockup period to reach 100% DLOM.



THE VFC DLOM CALCULATOR

Formula

Liquidity is the ability to sell quickly when the investor decides to sell. Privately held companies lack liquidity when compared to publicly traded companies. DLOM is the result of illiquidity. It represents the economic risk associated with failing to realize gains or failing to avoid losses on an investment during the period the investor is trying to sell it. The VFC DLOM calculator uses the Longstaff look-back model in calculating DLOM. The formula is:

$$Discount = V\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right)+V\sqrt{\frac{\sigma^2 T}{2\pi}}\exp\left(-\frac{\sigma^2 T}{8}\right)-V$$

where:

$V = current\ value\ of\ the\ investment$

$\sigma = volatility$

$T = marketability\ restriction\ period$

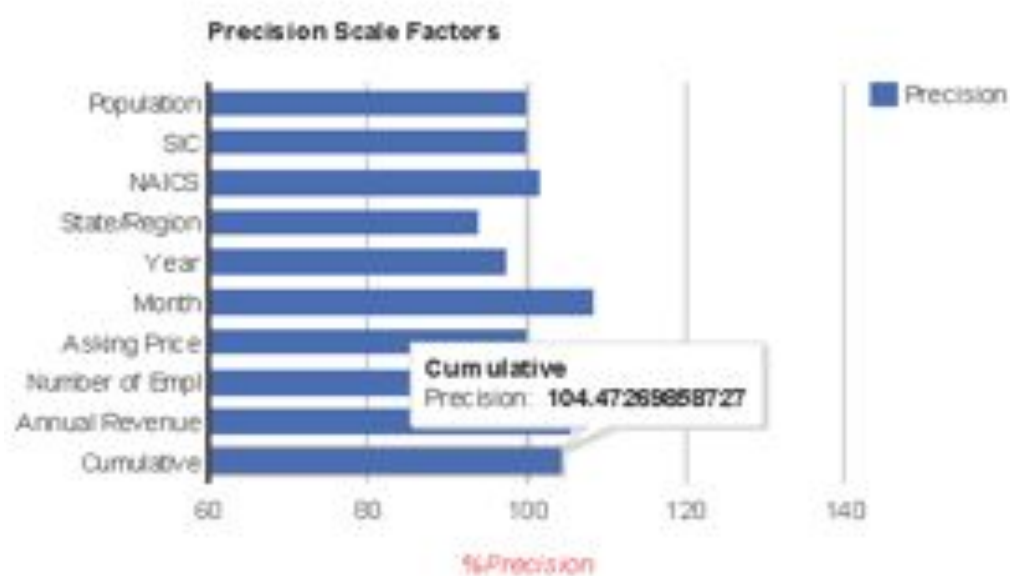
$N = standard\ normal\ cumulative\ distribution\ function$

Probability Estimates

The two variables that are required by the Longstaff look-back model are marketing period and price volatility. The marketing period is the time a business takes to sell from the date it is listed. The VFC Marketing Period Estimator gives the user a probability distribution of the marketing period.

It has been determined that that the marketing period of privately held business is influenced by industry, price, size, listing month, and listing year. The estimator uses data from BizComp® to calculate the mean and standard deviation marketing period based on the whole database and subsets of the population corresponding to the aforementioned influencers. Each subset has a number of sub-parameters, and each sub-parameter has an associated mean and standard deviation. If a single sub-parameter is selected, its mean and standard deviation are the basis for estimating the marketing period probabilities. If more than one sub-parameter is selected, the associated means and standard deviations are averaged, and the averages are the basis for estimating the marketing period probabilities.

The VFC DLOM Calculator provides a "Precision Engine" to assist the user in selecting parameters by visually showing whether a particular parameter is expanding or narrowing the distribution of marketing period data. Allowing the courser to hover over the bars of the Precision Engine displays the numerical value of the particular bar. The Precision Engine also provides a bar and numerical value for the cumulative effect of the selected parameters. Values greater than 100% are more precise than the distribution of the underlying data, and vice versa.



Once a mean and standard deviation are determined a statistical modeling engine transforms them into a log-normal probability distribution depicting the probability that the asset to be valued will sell within a certain length of time. An upper bound is applied to the probability distribution at the 95th percentile for statistical modeling purposes. The VFC Marketing Period Estimator will provide the mean, median, and mode of the distribution, probability graphs, and tables of the interval probabilities and the cumulative probabilities that support the graphs.

The second variable required by the Longstaff formula is price volatility. Price volatility is a measure of price risk faced by the seller of the business. The VFC Price Volatility Estimator works similarly to the VFC Marketing Period Estimator. The user can enter stock symbols for up to 20 guideline companies to use as benchmarks for volatility. The program will then calculate the mean and standard deviation of price volatility based on the companies provided by the user for look-back periods of 50 trading days, 100 trading days, 250 trading days, and 500 trading days. A statistical modeling engine then transforms the means and standard deviations into probability distributions depicting the probability that the asset to be valued will exhibit different volatility measures. An upper bound will be applied to the distribution at the point where the asset is 95% likely to have a volatility less than or equal to the value. The user will receive the mean, median, and mode of the probability distributions, probability graphs, and tables of the interval probabilities and cumulative probabilities that support the graphs for each of the different look-back periods.

Calculating DLOM

In calculating DLOM, the user can choose to use the VFC methodology for estimating marketing period and price volatility, or they may enter one or both manually. If the user chooses the VFC methodology for estimating marketing period, the calculator will use the probability distribution described above in its calculation of DLOM. To calculate DLOM, the probability distribution is divided into marketing period segments that correlate with each cumulative percentage point of the probability depicted by the probability distribution. The time segments are not uniform. If the user chooses the VFC methodology for estimating price volatility, the same will be done to the probability distribution for price volatility. This will create a double probability distribution. The DLOM for each marketing period and price volatility combination is calculated using the Longstaff look-back model. The DLOMs are next multiplied by the probability associated with each combination of marketing period and price volatility to produce a probability weighted DLOM. These probabilities are re-weighted to account for the 95% bound placed on both distributions so that the total probability is 100%. The probability weighted DLOMs for all the marketing period and price volatility combinations are summed to produce a double probability weighted DLOM for the asset.

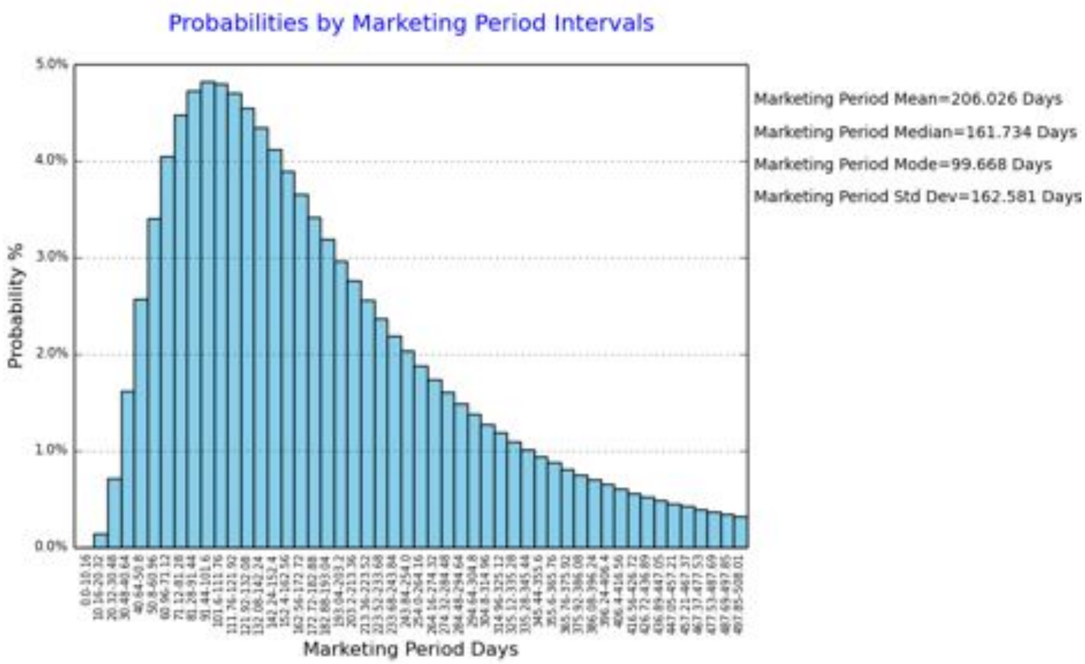
It is possible that certain values of price volatility will yield DLOMs that are greater than 100% for some transaction periods. These DLOMs will be limited to 100% for the calculation, as DLOM greater than 100% is not possible. These points will appear in red on the output graphs.

If the user chooses to use only the probability distribution to estimate marketing period or price volatility, single probability weighted DLOMs will be calculated. The DLOM will be calculated and probability weighted for each once percent probability segment of whichever variable is using the probability distribution. The other variable will remain will be fixed at the user-entered value for the DLOM calculation for each segment. The results are summed up to get the single probability weighted DLOM for the asset.

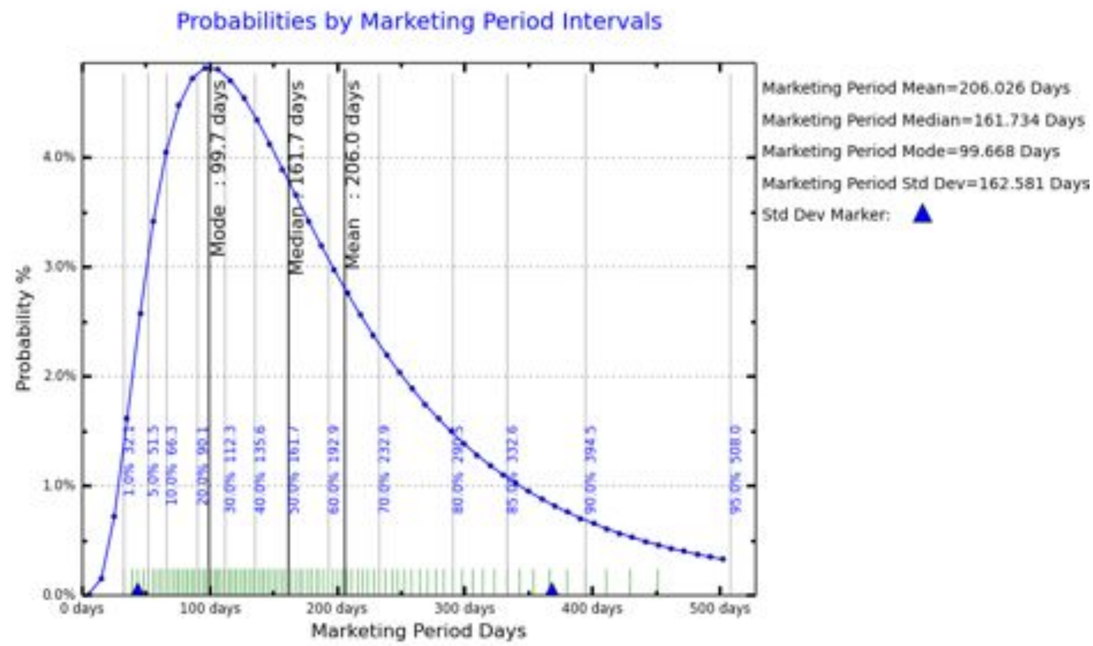
VFC Marketing Period Estimator Results:

Once the marketing period distribution is determined, the VFC Marketing Period Estimator provides several outputs. Use these tools to estimate the time needed to sell a business or other asset:

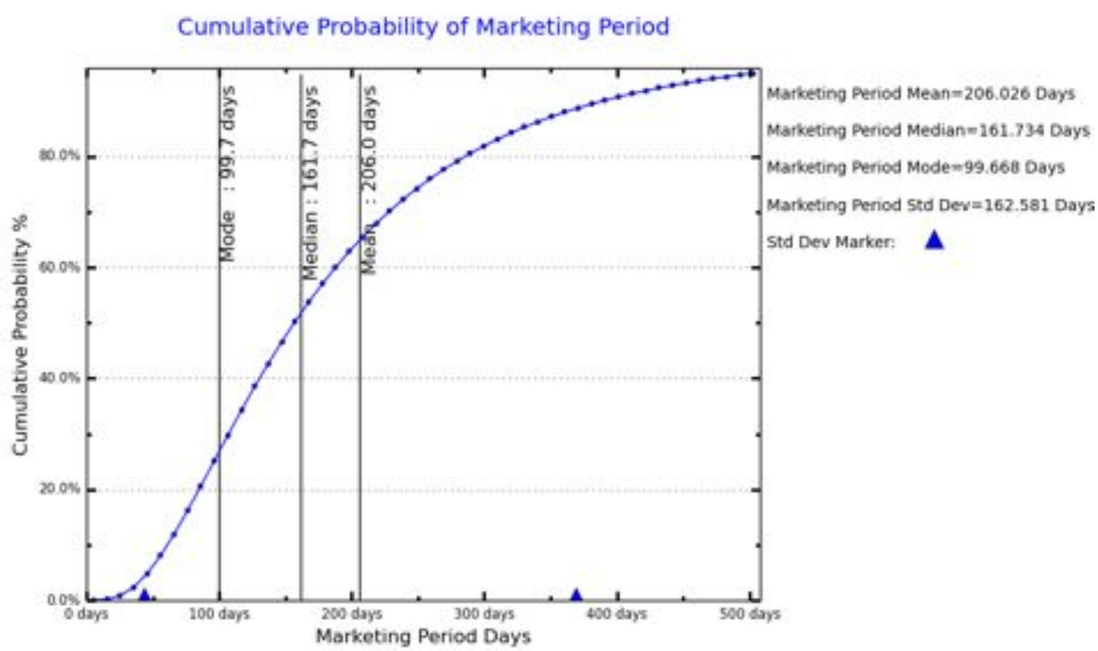
- A graph of the probability distribution of the marketing period intervals. This graph allows the user to estimate the probability of a sale occurring in a particular time period.



- A log normal probability density function graph showing the probability distribution of the marketing period. This graph also presents the mean, median, and mode of the range of marketing period probabilities, provides an indication of the relative concentration of anticipated sale events, and provides a means of estimating the percentage of sales events predicted to have occurred after a particular number of marketing days have elapsed.



- A graph of the increase in the cumulative probability of the marketing period as the marketing period increases. This graph also presents the mean, median, and mode of the range of marketing period probabilities.



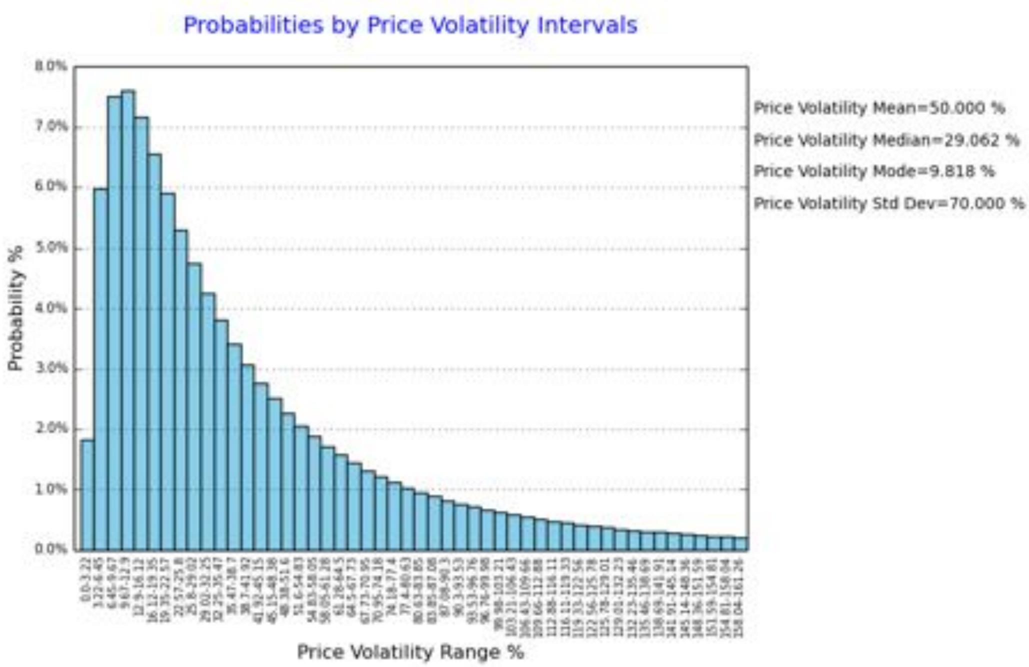
The VFC Marketing Period Estimator also provides downloadable tables of the values supporting the graphs, and a summary of the estimation factors and results:

- Mean and standard deviation of the VFC marketing period parameters selected by the user
- Mean and standard deviation of the marketing period parameters optionally provided by the user
- Adjusted mean marketing period and adjusted standard deviation used by the VFC Marketing Period Estimator, and the resulting median and mode marketing periods

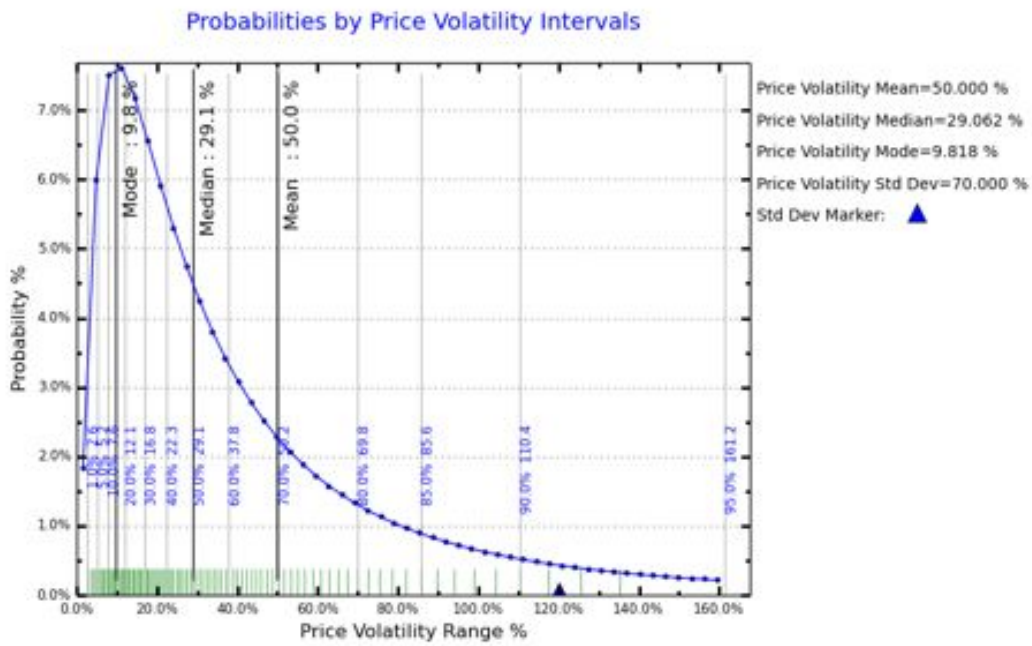
VFC Price Volatility Estimator Results:

The outputs provided by the VFC Price Volatility Estimator are similar to those provided by the VFC Marketing Period Estimator. Use these tools to estimate the price risk associated with a particular marketing period of an asset offered for sale or associated with the anticipated holding period of an asset:

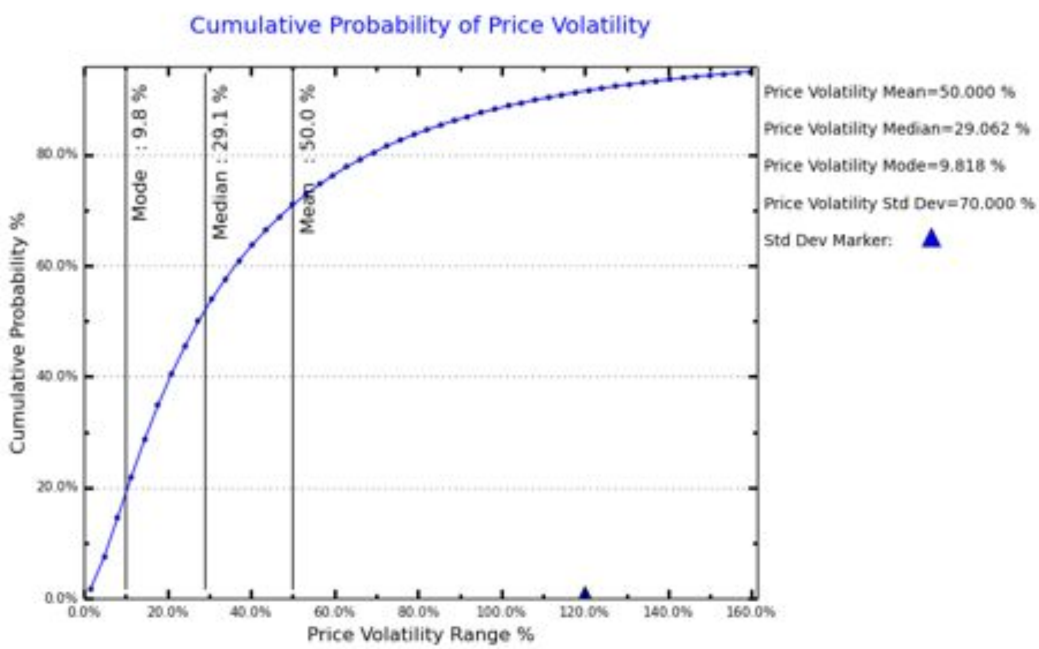
- A graph of the probability distribution of price volatility. This graph allows the user to estimate the probability of a particular range of price volatility.



- A log normal probability density function graph showing the probability distribution of the range of price volatility. This graph also presents the mean, median, and mode of the range of price volatility probabilities, and provides an indication of the relative concentration of potential volatility events.



- A graph of the increase in the cumulative probability of price volatility as the range of volatility increases. This graph also presents the mean, median, and mode of the range of price volatility probabilities.



The VFC Price Volatility Estimator also provides downloadable tables of the values supporting the graphs, and a summary of the estimation factors and results:

- Price volatility mean and standard deviation provided by the user

- Publicly traded companies that the user optionally selected as guidelines for price volatility estimation,
- Mean and standard deviation of the price volatility of each guideline company for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified
- Average mean and average standard deviation of the price volatility of the guideline company group for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified
- Mean, median, mode, and standard deviation of the price volatility probabilities for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified
- Time period that the user may have optionally specified for price volatility determination and the resulting mean, median, mode, and standard deviation
- Average mean and average standard deviation used by the VFC Price Volatility Estimator and the resulting price volatility median and mode.

<u>VFC Single Probability DLOM Calculator Results (Marketing Period Probability)</u>

The VFC Single Probability DLOM Calculator allows the user to provide a fixed price volatility assumption while using marketing period probability to calculate DLOM. In this application, the user receives outputs in addition to those described for the VFC Marketing Period Estimator. Use these tools to assess the quality of or to modify a DLOM estimate:

- A graph showing the cumulative probability-adjusted of DLOM as the marketing period increases. This graph also presents the mean, median, and mode of estimated DLOM over the range of marketing period probabilities.

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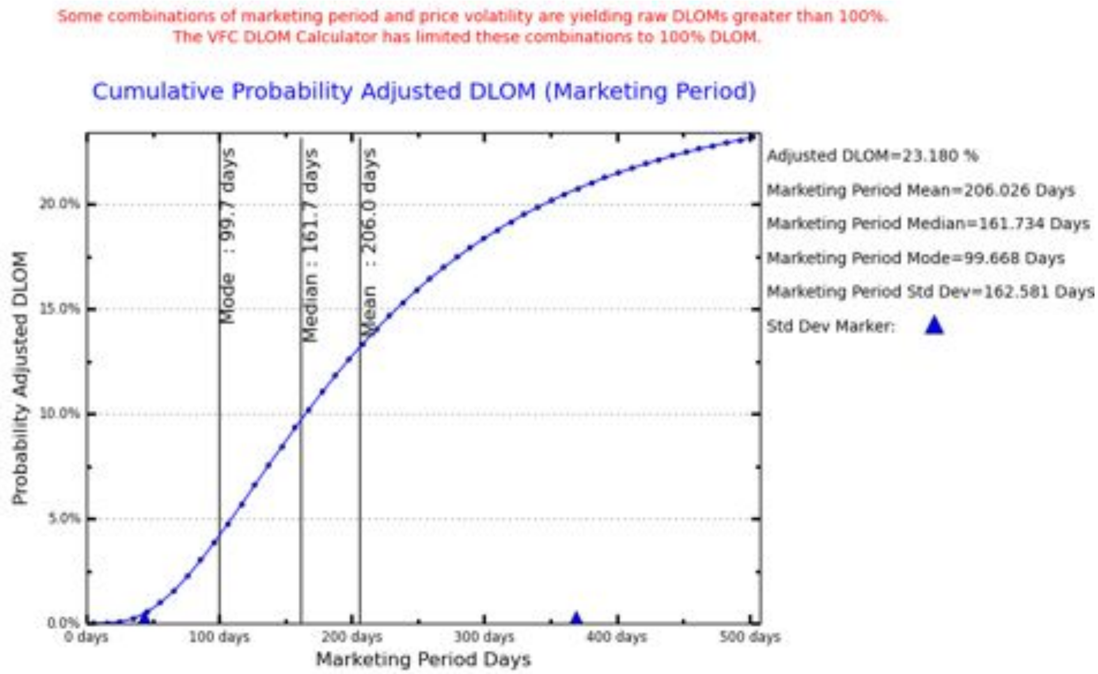
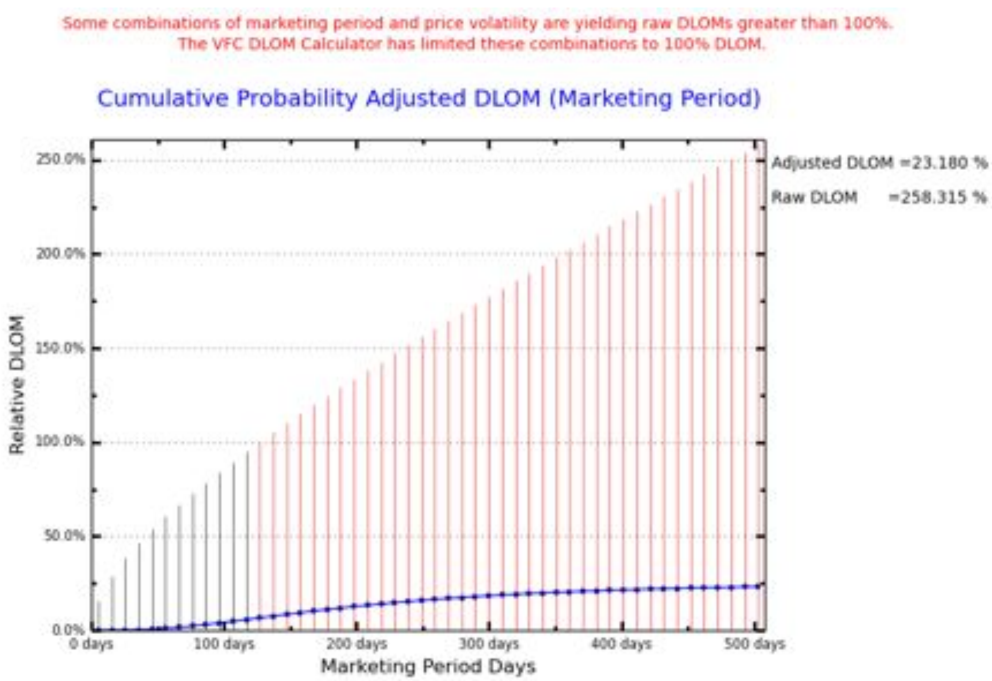


- A graph that compares (a) the cumulative growth of the VFC Probability-Based DLOM and (b) the raw Longstaff DLOM value that would result from applying the Longstaff formula for the particular marketing period without adjusting for the probability of occurrence or limiting the calculated DLOM to 100%. Marketing periods resulting in greater than 100% DLOM for a fixed volatility are shown in red.



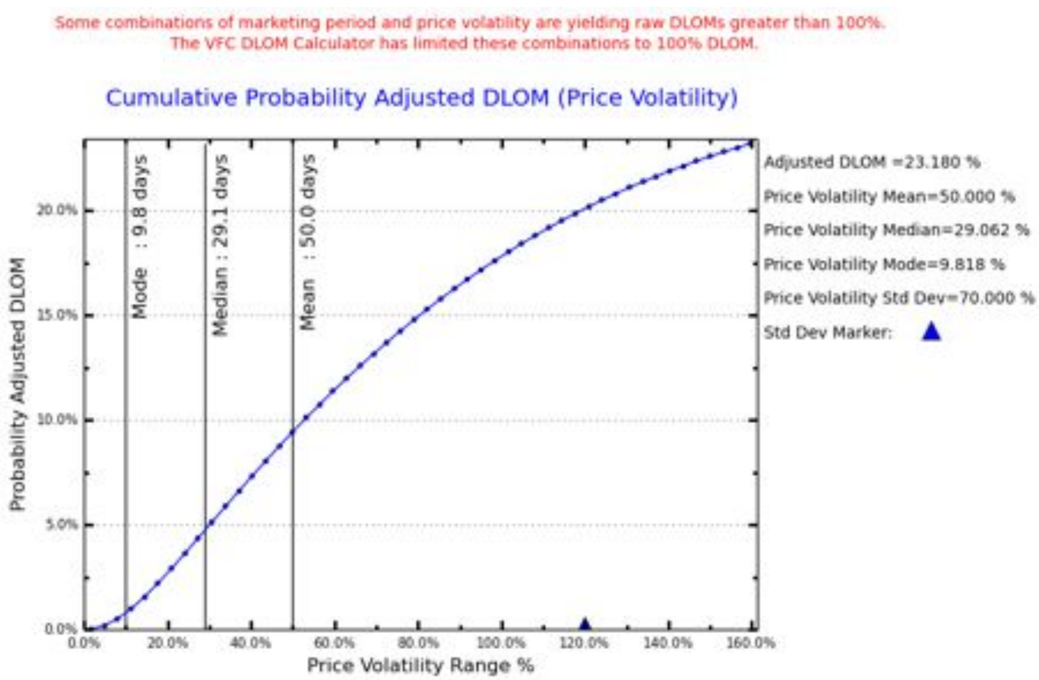
The VFC Single Probability DLOM Calculator (Marketing Period Probability) also provides downloadable tables of the values supporting the graphs, and a summary of the estimation factors and results:

- Mean and standard deviation of the VFC marketing period parameters selected by the user
- Mean and standard deviation of the marketing period parameters optionally provided by the user
- Adjusted mean marketing period and adjusted standard deviation used by the VFC Marketing Period Estimator, and the resulting median and mode marketing periods
- Price volatility assumption provided by the user
- Probability-weighted DLOM
- Raw Longstaff DLOM values

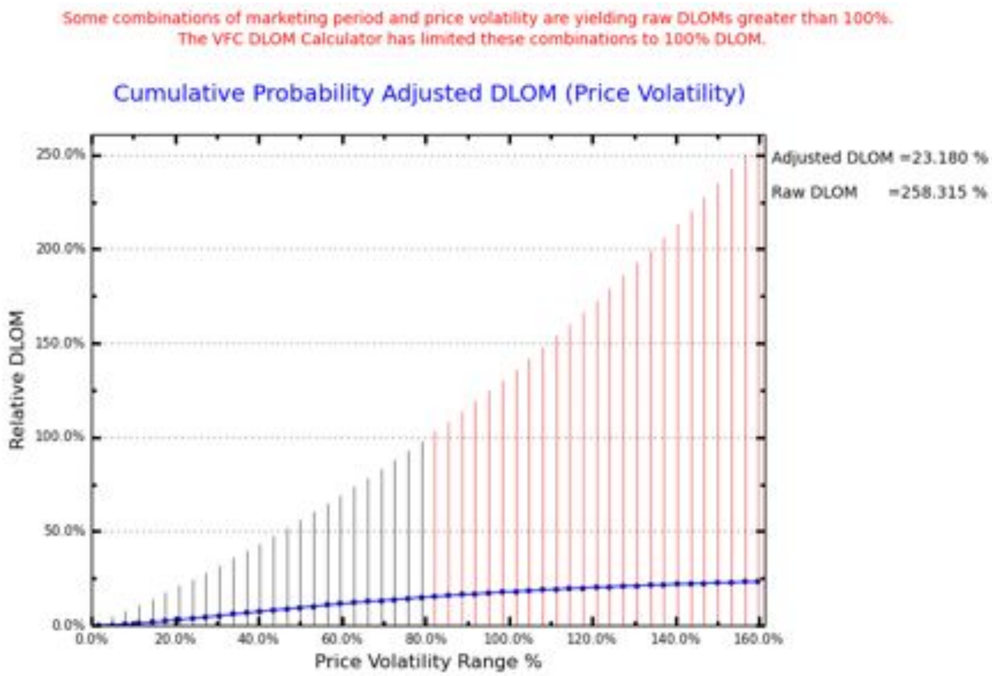
VFC Single Probability DLOM Calculator Results (Price Volatility Probability):

The VFC Single Probability DLOM Calculator also allows the user the option of fixing the marketing period assumption while using price volatility probability. In this application, the user receives outputs in addition to those described for the VFC Price Volatility Estimator. Use these tools to assess the quality of or to modify a DLOM estimate:

- A graph showing cumulative probability-adjusted DLOM as the price volatility increases. This graph also presents the mean, median, and mode of estimated DLOM over the range of price volatility probabilities.



- A graph that compares (a) the cumulative growth of probability adjusted, value limited DLOM and (b) the raw Longstaff DLOM values that would result from applying the Longstaff formula for the particular price volatility event without adjusting for the probability of occurrence or limiting the calculated DLOM to 100%. Price volatilities resulting in greater than 100% DLOM for a fixed marketing period are shown in red.



The VFC Single Probability DLOM Calculator (Price Volatility Probability) also provides downloadable tables of the values supporting the graphs, and a summary of the estimation factors and results:

- Price volatility mean and standard deviation provided by the user
- Publicly traded companies that the user optionally selected as guidelines for price volatility estimation
- Mean and standard deviation of the price volatility of each guideline company for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified
- Average mean and average standard deviation of the price volatility of the guideline company group for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified
- Mean, median, mode, and standard deviation of the price volatility probabilities for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified

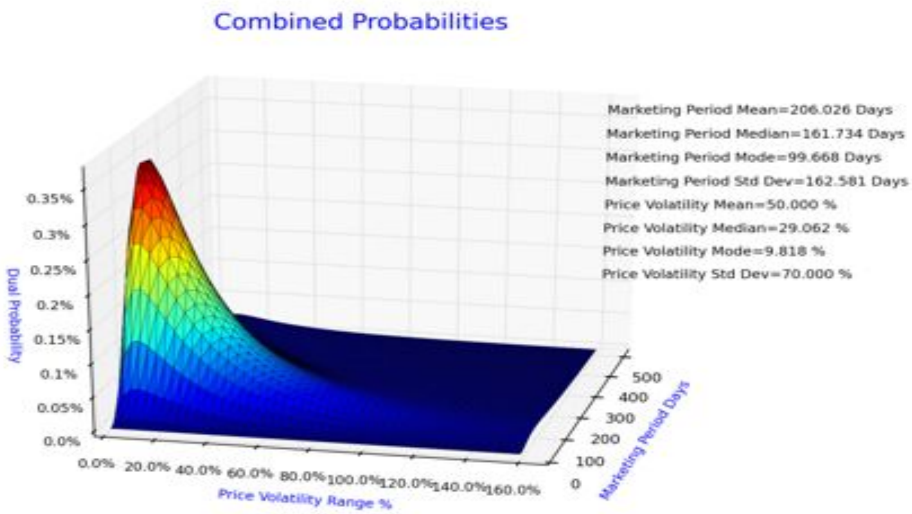
- Time period that the user may have optionally specified for price volatility determination and the resulting mean, median, mode, and standard deviation
- Average mean and average standard deviation used by the VFC Price Volatility Estimator and the resulting price volatility median and mode.
- Marketing period assumption provided by the user
- Probability-weighted DLOMs
- Raw Longstaff DLOM values

<u>VFC Double Probability DLOM Calculator Results (Marketing Period Probability and Price Volatility Probability):</u>

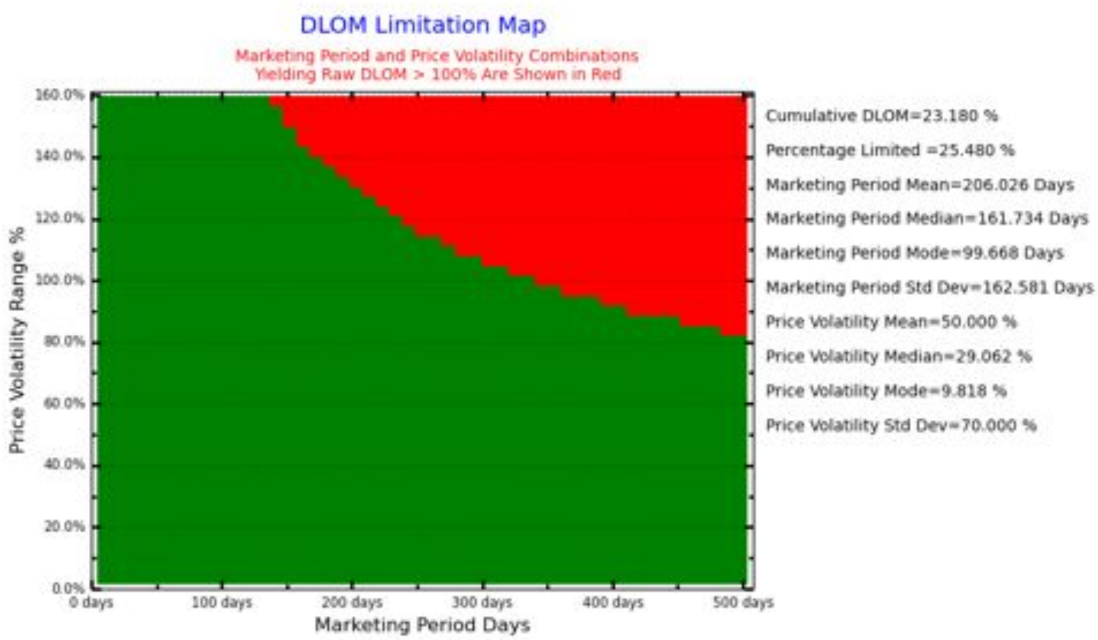
The VFC Double Probability DLOM Calculator allows the user the option of basing DLOM on the combined effects of marketing period and price volatility probabilities. In this application, the user receives outputs in addition to those described for the Single Probability DLOM Estimators and Calculators. Use these tools to assess the quality of or to modify a Double Probability DLOM estimate:

- A graph comparing the probability distributions of the predicted marketing periods and price volatilities. This graph allows the user to visualize the relative distribution of marketing period and price volatility probabilities.

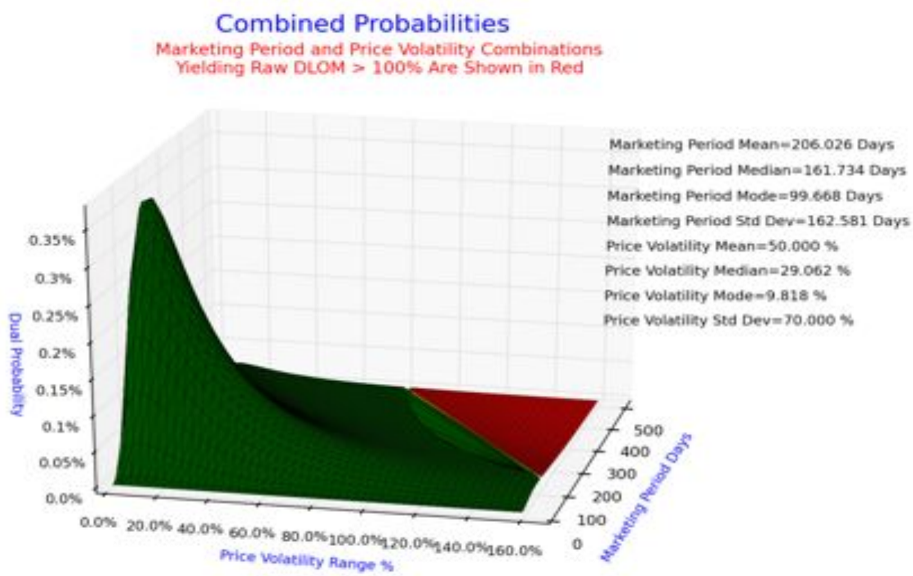
- A three-dimensional graph showing the distribution of the combinations of price volatilities and marketing periods. This graph allows the user to visualize the interaction of the determined marketing period and price volatility ranges of probability. This graph also reports the mean, median, mode, and standard deviations of the marketing period and price volatility probabilities.



- A 2-dimensional matrix graph of the probability events color coded in red to show events for which the raw Longstaff DLOM value exceeds 100%. This graph displays and reports the percentage of marketing period and price volatility combinations that result in raw Longstaff DLOM values greater than 100%. The VFC Double Probability DLOM Calculator limits such events to 100% DLOM.



- A 3-dimensional graph of the probability events color coded in red to show events for which the raw Longstaff DLOM values exceed 100%. This graph allows the user to visualize the overall influence of marketing period and price volatility combinations that have been limited by the VFC DLOM Calculator.



- A series of graphs that compare (a) the cumulative growth of probability adjusted and value limited DLOM over time measured at the mean, median, and mode of price volatility; and (b) the raw Longstaff DLOM value that would result from applying the Longstaff formula for the particular marketing period without adjusting for the probability of occurrence or limiting the calculated DLOM to 100%. Marketing periods resulting in greater than 100% DLOM for a fixed volatility are shown in red if they occur.

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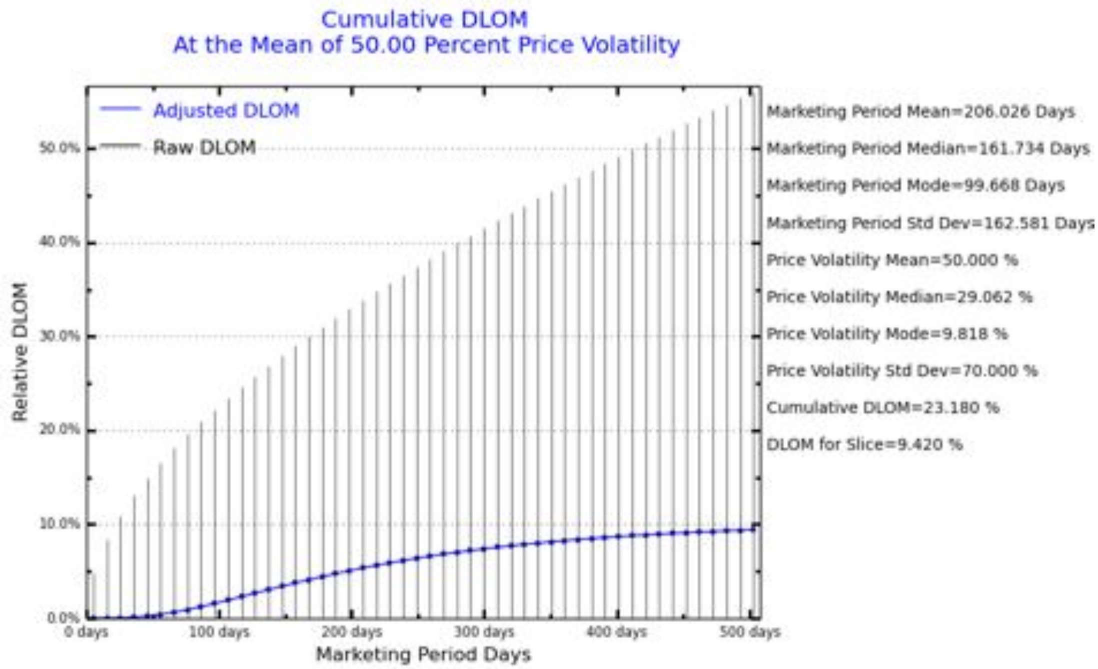
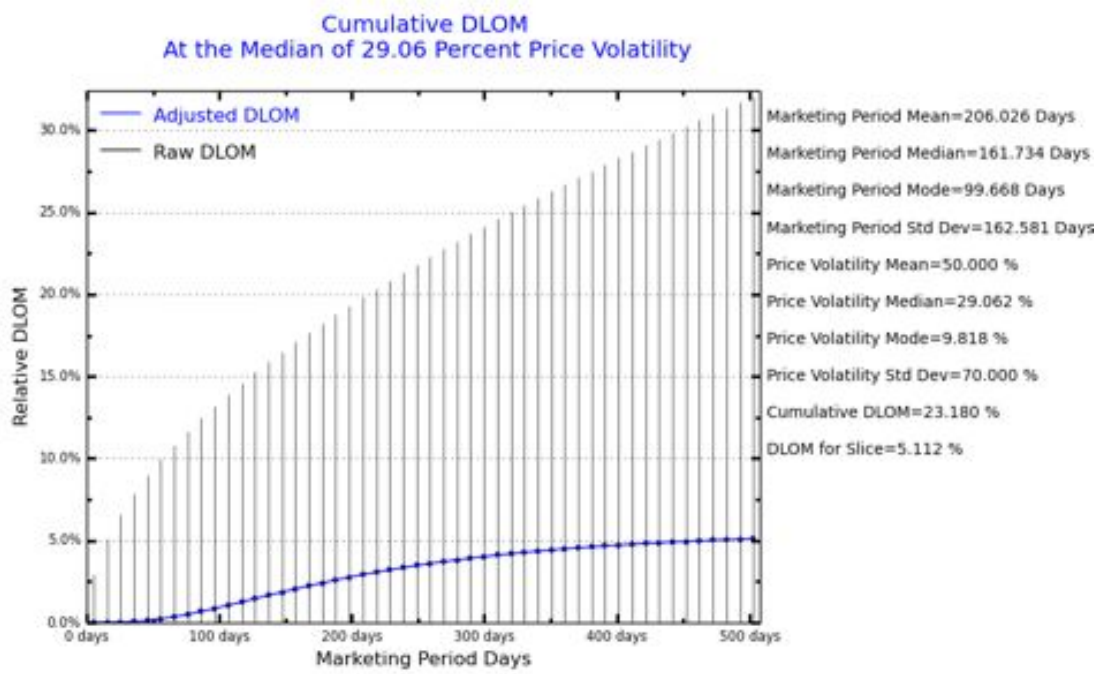

Cumulative DLOM
At the Mean of 50.00 Percent Price Volatility
Adjusted DLOM
Raw DLOM
Marketing Period Mean=206.026 Days
Marketing Period Median=161.734 Days
Marketing Period Mode=99.668 Days
Marketing Period Std Dev=162.581 Days
Price Volatility Mean=50.000 %
Price Volatility Median=29.062 %
Price Volatility Mode=9.818 %
Price Volatility Std Dev=70.000 %
Cumulative DLOM=23.180 %
DLOM for Slice=9.420 %
Relative DLOM
0.0%
10.0%
20.0%
30.0%
40.0%
50.0%
0 days
100 days
200 days
300 days
400 days
500 days
Marketing Period Days


Cumulative DLOM
At the Median of 29.06 Percent Price Volatility
Adjusted DLOM
Raw DLOM
Marketing Period Mean=206.026 Days
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Marketing Period Mode=99.668 Days
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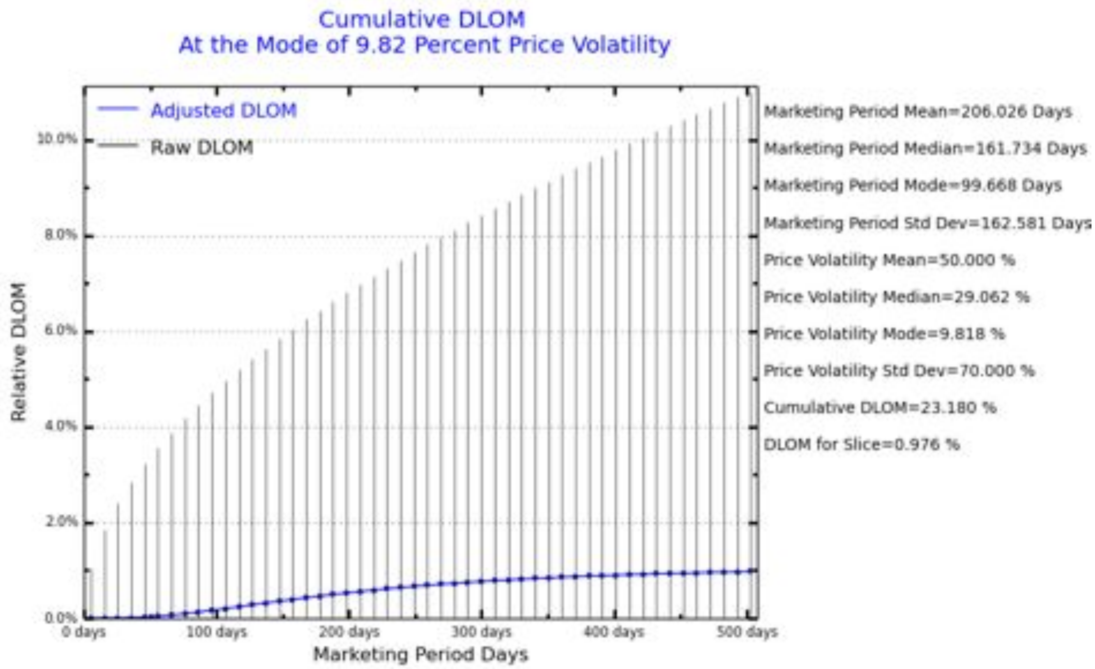
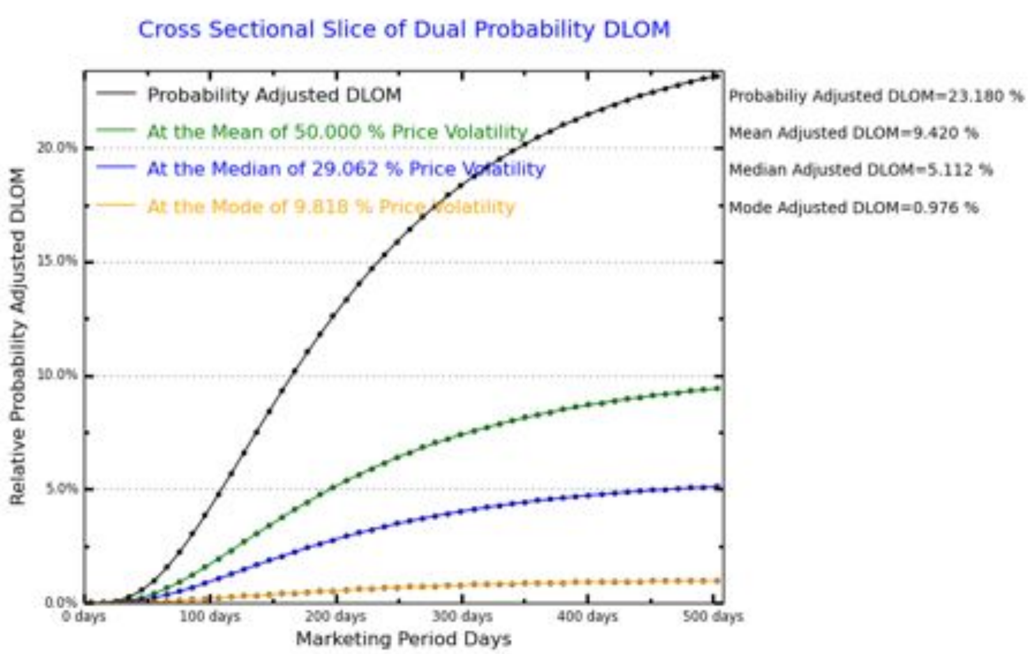
Price Volatility Mode=9.818 %
Price Volatility Std Dev=70.000 %
Cumulative DLOM=23.180 %
DLOM for Slice=5.112 %
Relative DLOM
0.0%
5.0%
10.0%
15.0%
20.0%
25.0%
30.0%
0 days
100 days
200 days
300 days
400 days
500 days
Marketing Period Days

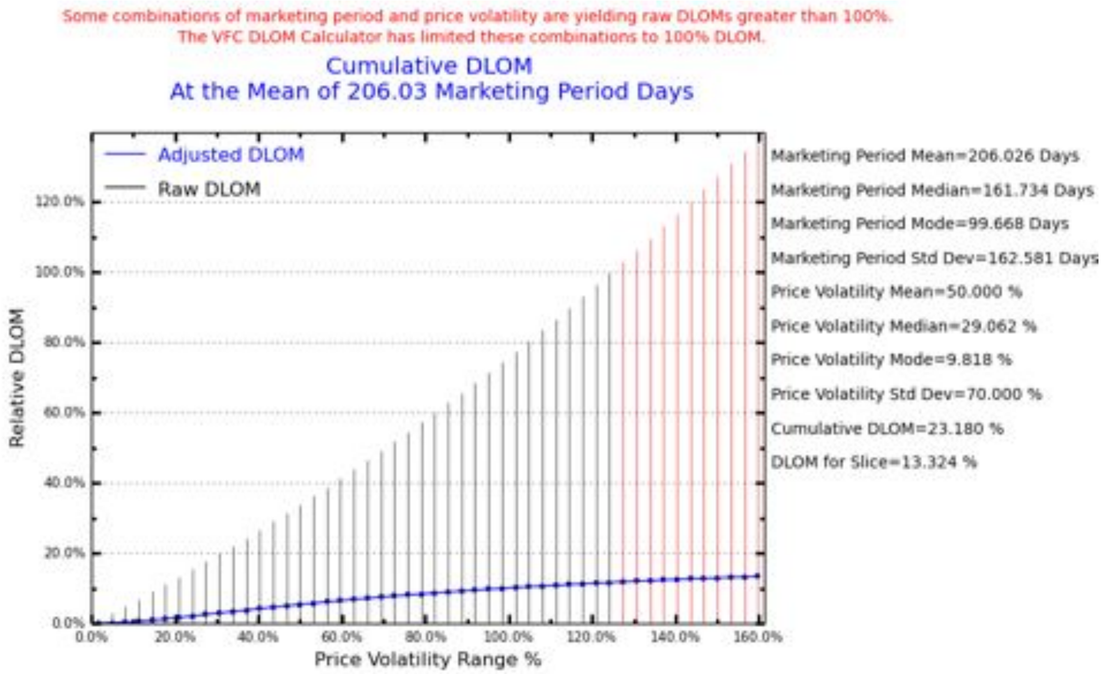


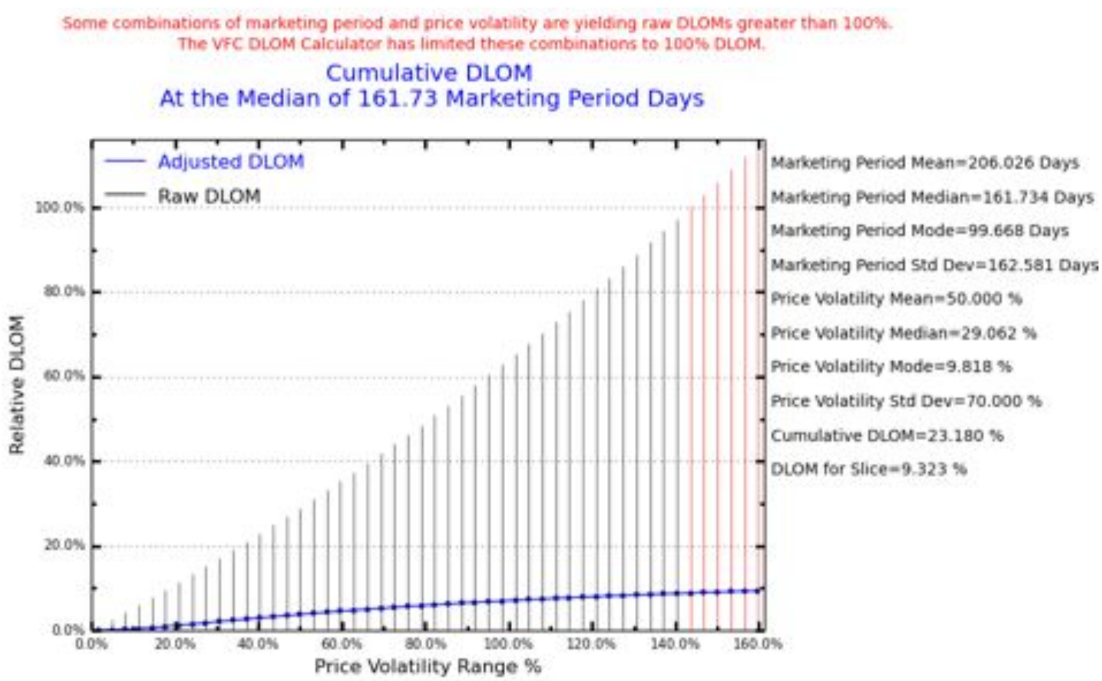
- A graph that compares the cumulative growth of DLOM measured at the mean, median, and mode of price volatility, and the cumulative growth of the VFC Probability-Based DLOM.

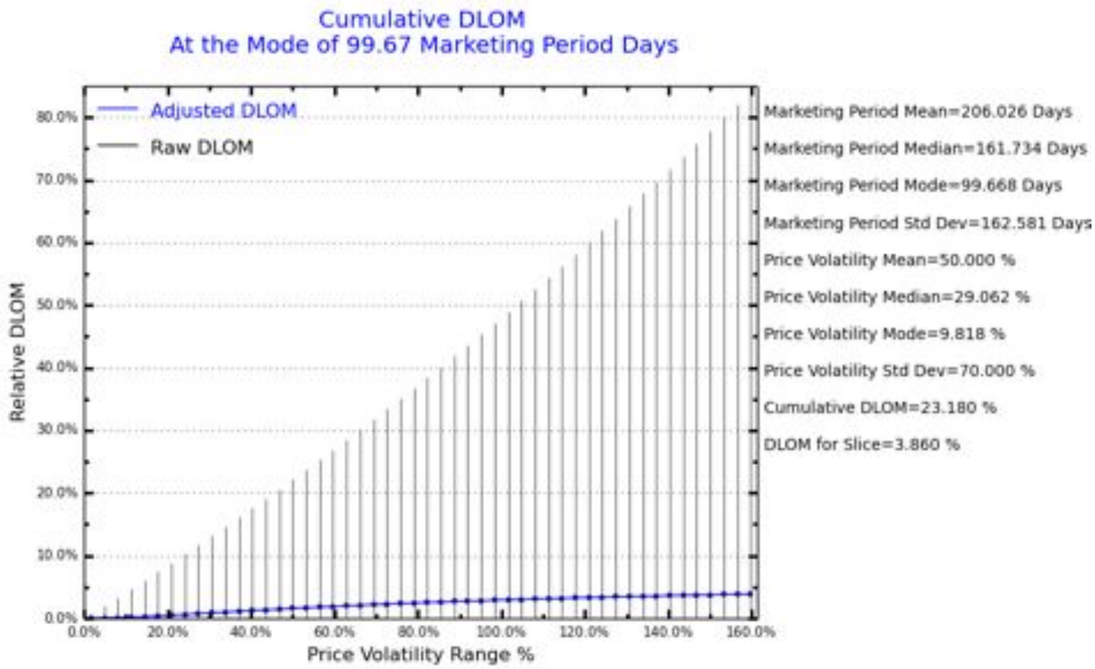


- A series of graphs that compare (a) the cumulative growth of probability adjusted and value limited DLOM over the predicted range of price volatility measured at the mean, median, and mode of marketing period probabilities; and (b) the raw Longstaff

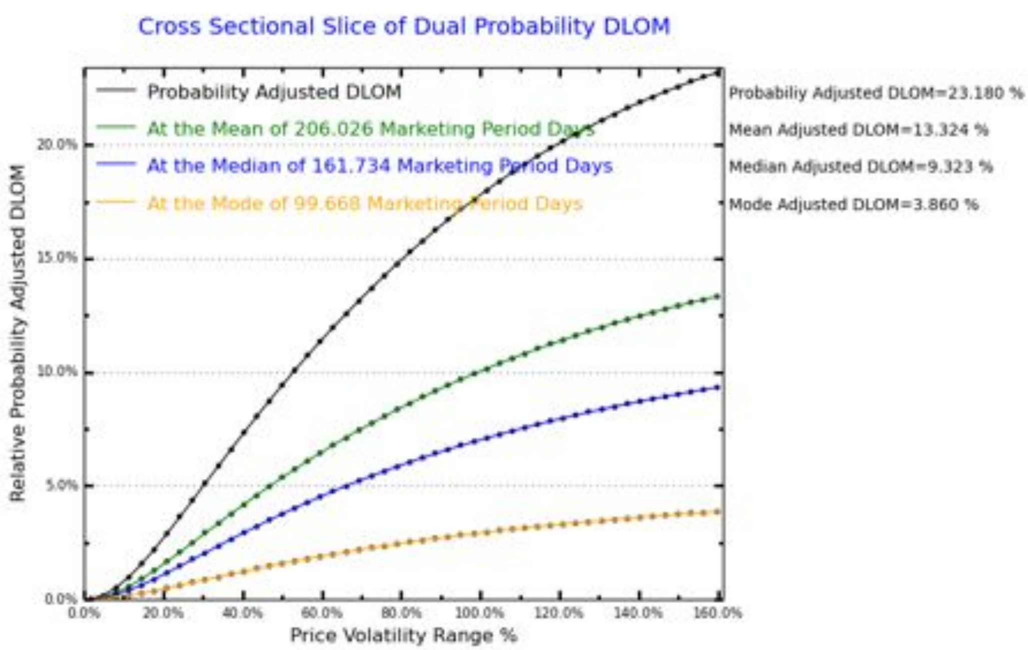
DLOM value that would result from applying the Longstaff formula for the particular price volatility without adjusting for the probability of occurrence or limiting the calculated DLOM to 100%. Price volatilities resulting in greater than 100% DLOM for a fixed marketing period are shown in red if they occur.



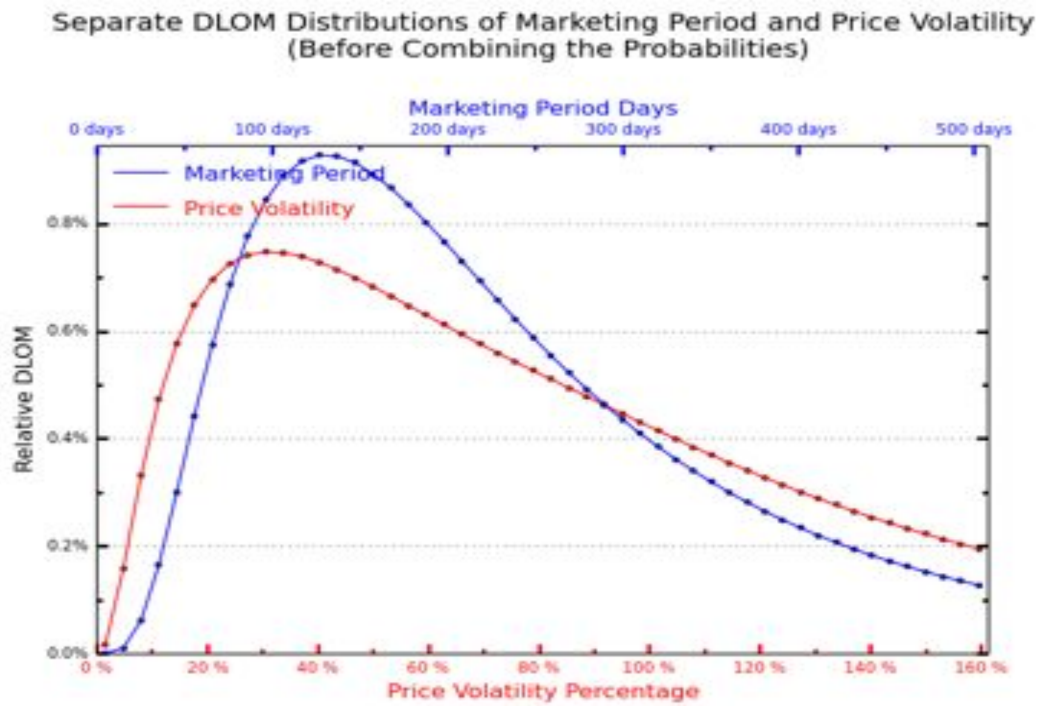




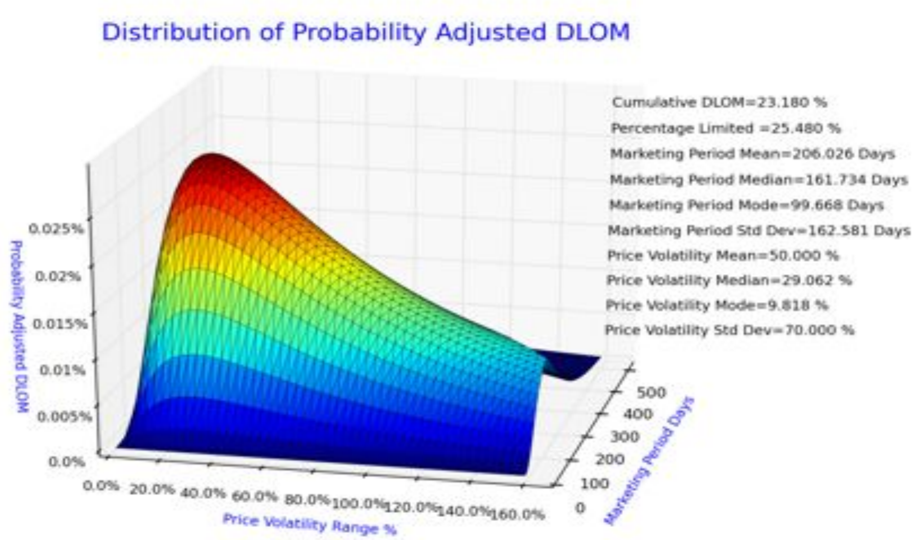
- A graph that compares the cumulative growth of the raw Longstaff DLOM measured at the mean, median, and mode of marketing period probabilities, and the cumulative growth of the VFC Probability-Based DLOM.



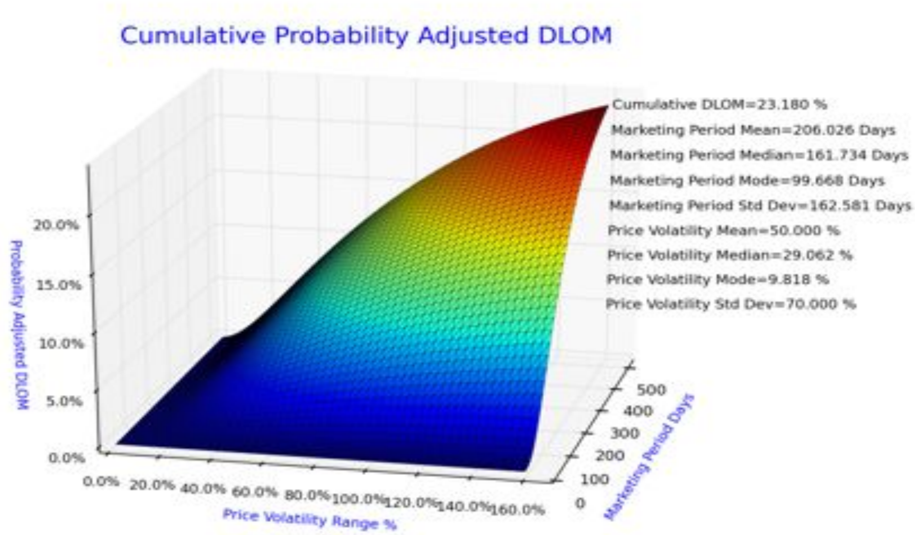
- A graph comparing the distributions of DLOM based on the predicted marketing period and price volatility trends. This graph allows the user to visualize the different influences of marketing period and price volatility on the VFC Probability-Based DLOM.



- A three-dimensional graph showing the distribution of the VFC Probability-Based DLOM. This graph allows the user to visualize how DLOM is influenced by the combination of the range of marketing period and price volatility probabilities.



- A three-dimensional graph showing cumulative growth of the double probability distribution of DLOM.



The VFC Double Probability DLOM Calculator also provides downloadable tables of the values supporting the graphs, and a summary of the estimation factors and results:

- Mean and standard deviation of the VFC marketing period parameters selected by the user
- Mean and standard deviation of the marketing period parameters optionally provided by the user
- Adjusted mean marketing period and adjusted standard deviation used by the VFC Marketing Period Estimator, and the resulting median and mode marketing periods
- Publicly traded companies that the user optionally selected as guidelines for price volatility estimation
- Mean and standard deviation of the price volatility of each guideline company for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified
- Average mean and average standard deviation of the price volatility of the guideline company group for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified
- Mean, median, mode, and standard deviation of the price volatility probabilities for 50-day, 100-day, 250-day, and 500-day price look back periods, and for a time period that the user may have optionally specified

- Time period that the user may have optionally specified for price volatility determination and the resulting mean, median, mode, and standard deviation
- Average mean and average standard deviation used by the VFC Price Volatility Estimator and the resulting price volatility median and mode.
- Probability-weighted DLOM.
- Mean, median, and mode of the probability-weighted DLOM.
- Adjusted and raw Longstaff DLOM values measured at the points of the mean, median, and mode of the marketing period and price volatility probabilities.
- Probability of each combination of marketing period and price volatility.
- Calculated DLOM for each combination of marketing period and price volatility probability.

DLOMs Specific to the Valuation Subject

Vianello Forensic Consulting, LLC recently launched the VFC DLOM Calculator to provide practitioners with a DLOM value that based on outcome probabilities as discussed above. The calculator delivers DLOMs that reflect the probability of each predicted combination of the marketing period and price volatility variables. In a manner that cannot be done with restricted stock and pre-IPO studies, practitioners can now craft DLOM conclusions that are specific to the valuation subject and the valuation date. Although the precise specifications applicable to a particular valuation engagement require the judgment of a highly skilled professional, the VFC DLOM Calculator aids the practitioner by (a) quickly and accurately making all of the necessary calculations, and (b) providing robust diagnostics to enhance analysis and communication to others.

The VFC DLOM Calculator provides easy-to-use drop down lists to tailor marketing periods specific to the valuation subject based on relevant factors of industry, seasonality, year, employee count, asking price, and revenues. Or the practitioner can enter his or her own marketing period metrics. Likewise, the VFC DLOM Calculator aids the practitioner by automatically calculating price volatilities and standard deviations for guideline companies and/or indices. Just enter the ticker symbols of the guidelines. Or the practitioner can enter his or her own price volatility metrics. Whether the marketing period variable or the price volatility variable, simply provide your inputs and the VFC DLOM Calculator will do the rest of the work.

The VFC DLOM Calculator is available at www.dlomcalculator.com. Vianello Forensic Consulting, LLC and the author are available for consultation, training, and continuation education seminars regarding the theories and methodologies underlying probability-based DLOM, and application of the VFC DLOM Calculator.

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vianello@vianello.biz

Vianello Forensic Consulting, LLC
May 14, 2014