

How Probability Affects Discounts for Lack of Marketability

By Marc Vianello, CPA, ABV, CFF

Editor's note: This article presents a methodology for determining DLOM that combines probability-based time and price volatility variables in conjunction with the formula put forward by Francis Longstaff, Ph.D. The author has also developed a DLOM Calculator that embodies this methodology. We welcome reader feedback, so please email your comments to the editor at andyd@bvresources.com.

Traditional methods for determining a discount for lack of marketability (DLOM) fall short for several reasons. Restricted stock and pre-IPO studies do not have a sufficient number of transactions, and they do not reflect "price risk." Price risk is the time it takes to sell an asset that is not publicly traded.

Price risk is influenced by liquidity, which represents the ability to sell an investment quickly when the investor decides to sell. Conversely, lack of liquidity represents the cost of failing to realize gains or failing to avoid losses associated with an investment during the period in which the asset cannot be sold or is being offered for sale. With that understanding, DLOMs should reflect the price risk of the investment during the period of time that it is expected to be marketed for sale.

In the past, estimating DLOM has effectively represented a guess by the appraiser based on the empirical results of studies of discounts observed with restricted stocks and initial public offerings. Such studies have been used to quantify DLOM since the early 1970s. But the studies

offered little reliability because of limited sample sizes. In recent years, commercially offered restricted stock and pre-IPO studies have sought to comprehensively include the relevant transactions in the marketplace, but nonetheless they are constrained by an insufficient number of available transactions. Also, they do not easily reconcile to the price risk associated with the time it takes to sell an asset that is not publicly traded. The result is that the commercially offered restricted stock, pre-IPO studies, and databases do not offer appraisers transactional data that are sufficiently comparable to subject companies and valuation dates and do not offer DLOM answers that are sufficiently comprehensive, objective, and accurate to provide defensible DLOM estimates.

Look-back option DLOMs. In 1995, Dr. Francis A. Longstaff suggested that the look-back option pricing model could be used to calculate the DLOM for a privately held company.¹ The variables that enter into what has become known as the "Longstaff formula" are: (1) the time period of a restriction on marketability; and (2) the price volatility during the restriction period.

In 2009, the IRS published a broad discussion of the extant methodologies for estimating DLOMs in a document called "Job Aid for IRS Valuation Professionals" in 2009 (IRS Job Aid). The IRS Job Aid concluded, in part:

¹ Francis A. Longstaff, "How Much Can Marketability Affect Security Values?" *The Journal of Finance*, Vol. L, No. 5, 1995, pp. 1767-74.

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The Longstaff approach assumes perfect market timing and, therefore, derives an upper bound for the lack of marketability discount since an investor is looking backward in time to make his buy/sell decisions instead of making these decisions based on present evidence and anticipated future stock price movements. Volatilities in excess of 30% would most likely be used as a proxy for privately held stock for which there is no public market. Therefore, the model may produce results which are not realistic as the upper bound in circumstances of this kind could well reach 100%.²

This author later refined understanding of the Longstaff formula with a number of observations.³ First, the time period variable should be equated with the period of time that the asset is expected to be offered for sale, that is, a forward-looking time period commencing with the valuation date. Second, the price volatility variable should be equated with that expected marketing period of the asset, also making it forward looking. Third, “perfect timing” exists in the valuation context because it always equates to the appraisal date. Fourth, using average expected price volatility puts price risk within the reasonable expectation of the average investor, eliminating a supposed need for “perfect knowledge.” Fifth, capping at 100% the DLOMs calculated at the farther reaches of time and price volatility eliminates this criticism. These refinements aligned the Longstaff formula to the need to base DLOMs on the price risk associated with the period of

2 Internal Revenue Service, *Job Aid for IRS Valuation Professionals*, Sept. 25, 2009, page 33.

3 Marc Vianello, “New Insight Into Calculating Discounts for Lack of Marketability,” *Financial Valuation and Litigation Expert*, Issue 11, February/March 2008 and *CPA Expert*, American Institute of Certified Public Accountants, May 2008. Marc Vianello, “Calculating DLOM Using the VFC Longstaff Methodology,” *BVR’s Guide to Discounts for Lack of Marketability*, 2009 Edition. Marc Vianello, “The Marketing Period of Private Sales Transactions,” *Business Valuation Update*, Vol. 16, No. 12, December 2010. Marc Vianello, “Rebutting Critics of the Longstaff DLOM Methodology,” *Business Valuation Update*, Vol. 18, No. 9, September 2012.

time that an asset cannot be sold or is being held explicitly for sale.

The IRS Job Aid also provided a table showing Longstaff-based DLOMs calculated over ranges of time periods and price volatilities and contended that volatilities in excess of 30% are “not realistic” for estimating DLOM using look-back option pricing models. In support, the IRS Job Aid provided a table (summarized in Exhibit 1) reporting marketability discounts in excess of 100% resulting from combinations of variables of at least 50% volatility with a five-year marketing period and at least 70% volatility with a two-year marketing period.

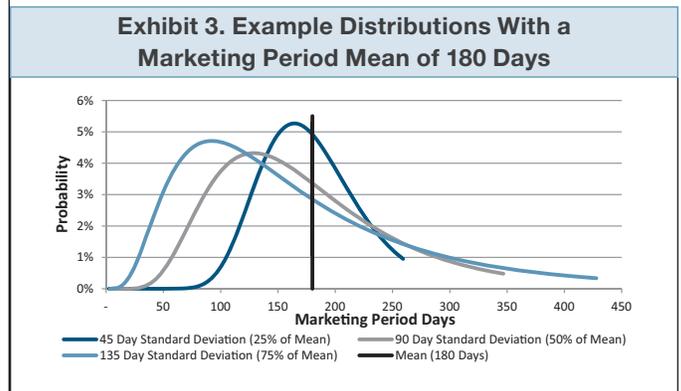
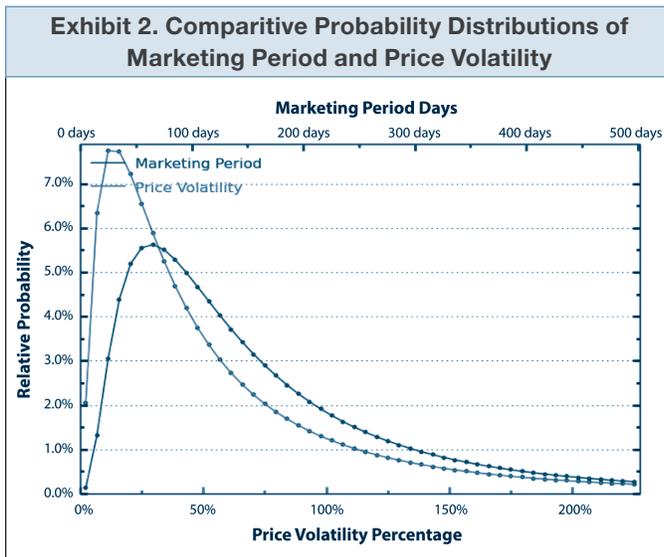
It is obvious that if the DLOMs shown by the IRS were simply limited by practitioners to 100%, then the criticism associated with the two-year/70% and five-year/70% values shown in Exhibit 1 would be eliminated in every similar instance.

Using a static time period and/or static price volatility in the Longstaff formula as the IRS did can be appropriate in situations where either or both of those variables are certain. For example, one might want to calculate the DLOM for an asset that will be sold on a certain date. But price is rarely a constant and usually has a range of probabilistic outcomes. However, asset marketing periods of like assets are rarely a constant, and asset prices typically exhibit mean volatilities greater than 30% and substantial underlying standard deviation.

The solution for the appraiser is to base DLOM conclusions on a probability-based approach that accounts for the full range of predicted outcomes. Price volatilities and nonconstant asset marketing periods exhibit lognormal distribution tendencies since the values cannot be less than zero. Typical patterns are shown in Exhibit 2.

The effects of standard deviation on probability distributions. Standard deviation is a statistical measure of how dispersed data points are from the mean. Standard deviations increase as the underlying population becomes more dispersed and vice versa. A lower standard deviation signifies that the data points tend to be gathered closer to the statistical mean. Thus, distributions with relatively low standard deviations exhibit modes (the point designated by the peak of the distribution) that are relatively closer to the population mean. And distributions with relatively high standard deviations exhibit modes that are relatively farther from the population mean. With a lognormal distribution, these attributes cause the modes of high standard deviation distributions

Exhibit 1. Raw DLOM			
Marketing Period Days	Raw DLOM at Price Volatility		
	10%	40%	70%
30	2.3%	9.5%	17.0%
180	5.7%	24.5%	45.7%
365	8.2%	36.1%	69.2%
730	11.8%	53.7%	106.7%
1,825	19.1%	93.7%	198.5%



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to be closer to zero than the modes of low standard deviation distributions as shown in Exhibit 3. Each of the distributions shown in the exhibit has a mean of 180 days but a different standard deviation: (a) the dark blue line has a standard deviation of 45 days (25% of the mean); (b) the gray line has a standard deviation of 90 days (50% of the mean); and (c) the light blue line has a standard deviation of 135 days (75% of the mean).

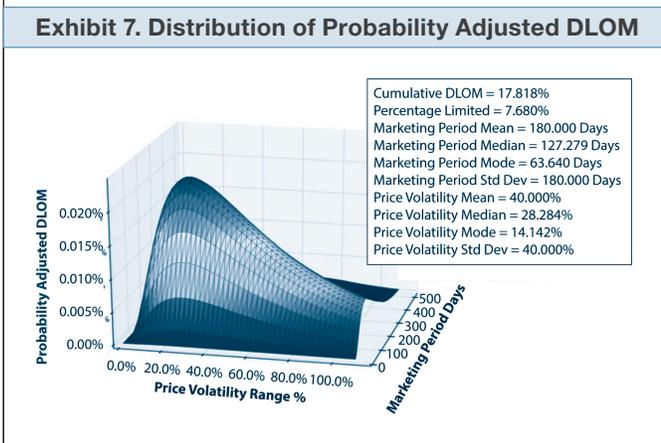
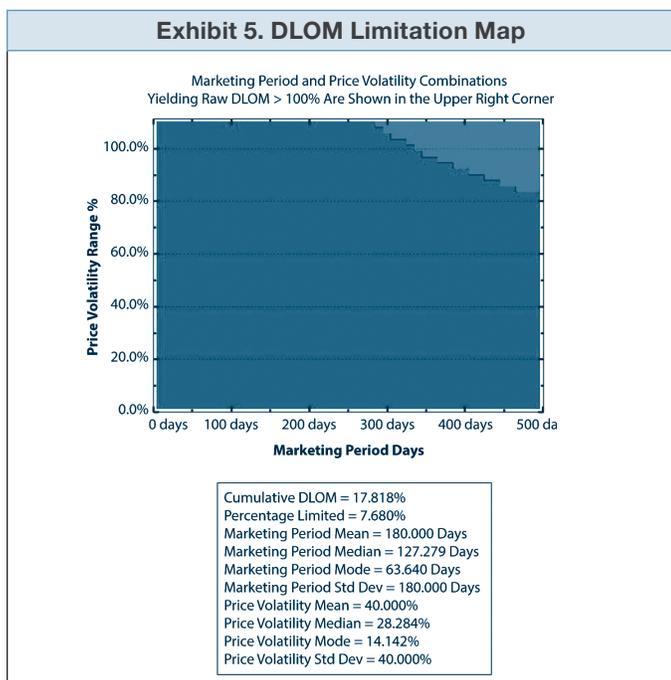
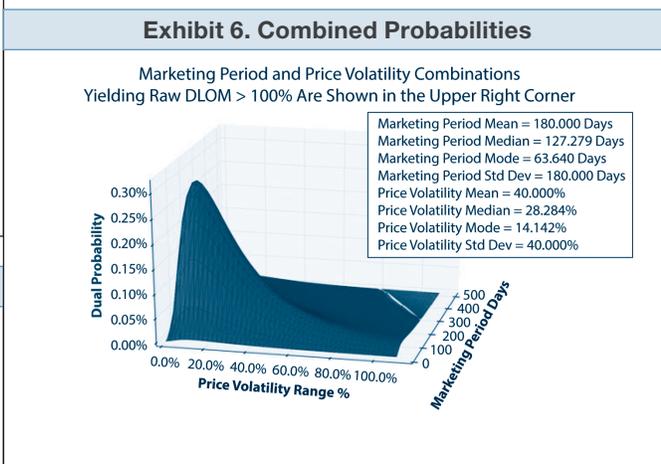
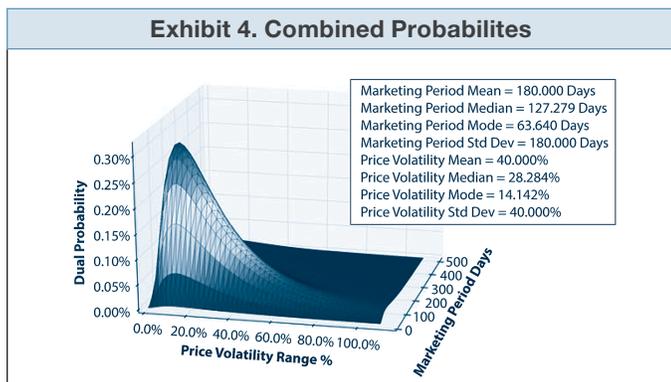
Adding probability to the Longstaff formula.

Envision a population of asset sale transactions with a mean marketing period of 180 days and a standard deviation of 180 days. Now envision that the price risk associated with the population of assets has a price volatility mean of 40% and a standard deviation of 40%. Calculating DLOM

over the range of either of the distributions would yield a value that reflects the full range of the separate likely outcomes. The combined probabilities would look like Exhibit 4, with the preponderance of likely outcomes concentrated around the combined modes of the distributions of the two variables. It is readily seen that the relative chance of greatly extended marketing periods and very high price volatilities in the envisioned scenario is remote.

Despite low probability of occurrence, the extreme combinations of marketing period and price volatility can result in points for which "raw" DLOMs exceed 100%. Exhibits 5 and 6 show where such points occur in the combined distributions.

After limiting raw DLOMs to 100%, the Vianello Forensic Consulting (VFC) DLOM Calculator weights each raw DLOM by its probability of



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occurrence. In this example, the result would be a DLOM distribution as shown in Exhibit 7, with a resulting DLOM conclusion of 17.8% instead of the 24.5% raw DLOM shown in Exhibit 1.

Single probability DLOM. Now let's create some probability-based alternatives to Exhibit 1. First, assume that the price volatilities of Exhibit 1 are static values, the marketing periods have standard deviations equal to 50% of their means, and raw DLOMs in excess of 100% are limited to 100%. The resulting single-probability DLOMs are presented in Exhibit 8.

Alternatively, assume that the standard deviations of the marketing periods are equal to 200% of their means. The resulting single-probability DLOMs are presented in Exhibit 9.

Comparing Exhibits 8 and 9 reveals the effects of different probability assumptions on the different combinations of marketing period and price volatility. Contrary to intuition, larger standard deviations result in smaller DLOMs because increased

skewing of the distribution away from the mode places more events into the lower probabilities of the range of outcomes.

Now let's re-create Exhibit 1 assuming that the marketing periods are static, the price volatilities have standard deviations equal to 50% of their means, and raw DLOMs have been limited to 100%. The resulting single-probability DLOMs are presented in Exhibit 10.

Alternatively, assume that the standard deviations of the price volatilities are equal to 200% of their means. The resulting single-probability DLOMs are presented in Exhibit 11.

As with the marketing period variable, comparison of Exhibits 10 and 11 reveals that larger standard deviations of price volatility result in correspondingly smaller DLOMs. This is because of the same skewing effect.

Comparisons of Exhibits 8 and 10 and Exhibits 9 and 11 reveal another aspect of probability-based

Exhibit 8. DLOM Adjusted for Marketing Period Probability				
Marketing Period Days		Probability Adjusted DLOM at Price Volatility		
Mean		10%	40%	70%
	Std Dev			
30	15.0	2.2%	8.9%	16.0%
180	90.0	5.4%	23.0%	42.9%
365	182.5	7.8%	34.0%	65.0%
730	365.0	11.1%	50.5%	89.3%
1,825	912.5	18.0%	83.2%	99.7%

Exhibit 10. DLOM Adjusted for Price Volatility Probability				
Marketing Period Days		Probability Adjusted DLOM at Price Volatility		
Mean		10%	40%	70%
	Std Dev	5%	20%	35%
30		2.1%	8.8%	15.9%
180		5.3%	22.8%	42.8%
365		7.6%	33.8%	61.7%
730		11.0%	50.0%	79.7%
1,825		17.8%	74.8%	94.7%

Exhibit 9. DLOM Adjusted for Marketing Period Probability				
Marketing Period Days		Probability Adjusted DLOM at Price Volatility		
Mean		10%	40%	70%
	Std Dev			
30	60	1.7%	6.9%	12.3%
180	360	4.2%	17.7%	32.9%
365	730	6.0%	26.1%	47.7%
730	1,460	8.5%	38.4%	63.7%
1,825	3,650	13.8%	59.0%	82.3%

Exhibit 11. DLOM Adjusted for Price Volatility Probability				
Marketing Period Days		Probability Adjusted DLOM at Price Volatility		
Mean		10%	40%	70%
	Std Dev	20%	80%	140%
30		2.1%	8.8%	15.9%
180		5.3%	22.8%	42.8%
365		7.6%	33.8%	61.7%
730		11.0%	50.0%	79.7%
1,825		17.8%	74.8%	94.7%

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DLOM. One might anticipate that toggling the input value of the marketing period and price volatility variables would result in the same DLOMs. It does not because the Longstaff formula squares price volatility but does not square time. This magnifies the effects of changes in price volatility assumptions relative to changes in time period assumptions.

Double-probability DLOM. Now let's consider how DLOM is affected by combining the inputs of Exhibits 8 and 10 and Exhibits 9 and 11. A double-probability scenario involving dual 0.5 coefficients of variation is shown in Exhibit 12. With this low standard deviation assumption, there is a slight further diminution of the resulting DLOMs.

Alternatively, a double-probability scenario using dual 2.0 coefficients of variation is shown in Exhibit 13. In contrast to Exhibit 12, however, this high standard deviation scenario demonstrates a substantial diminution of the resulting DLOMs.

Applying probability to both the time and price volatility variables of the Longstaff formula reduces DLOM relative to corresponding single-probability calculations. This result occurs because the compounding effect of two probability functions further skews the distribution of likely outcomes. The downward effect on DLOMs becomes greater as standard deviations increase.

DLOMs specific to the valuation subject. My firm, Vianello Forensic Consulting LLC, recently launched the VFC DLOM Calculator to provide practitioners with a DLOM value that is based on outcome probabilities as discussed above. The calculator delivers DLOMs that reflect the probability of each predicted combination of the marketing period and price volatility variables. In a manner that cannot be done with restricted stock and pre-IPO studies, practitioners can now craft DLOM conclusions that are specific to the valuation subject and the valuation date. Although the precise specifications applicable to a particular valuation engagement require the judgment of a highly skilled professional, the VFC DLOM Calculator aids the practitioner by: (a) quickly and accurately making all of the necessary calculations; and (b) providing robust diagnostics to enhance analysis and communication to others.

The VFC DLOM Calculator provides easy-to-use dropdown lists to tailor marketing periods specific to the valuation subject based on relevant factors of industry, seasonality, year, employee count, asking price, and revenues, or the practitioner can enter his or her own marketing period metrics. Likewise, the VFC DLOM Calculator aids the practitioner by automatically calculating price volatilities and standard deviations for guideline companies and/or indices. Just enter the ticker symbols of the guidelines, or the practitioner can enter his or her own price volatility metrics. Whether the marketing period variable or the price volatility variable, simply provide your inputs and the VFC DLOM Calculator will do the rest of the work.

Free trial. *BVU* subscribers have an exclusive opportunity to test-drive the VFC DLOM Calculator during the month of July 2014. Go to www.dlomcalculator.com and enter the promo

Exhibit 12. DLOM Adjusted for Marketing Period and Price Volatility Probabilities				
Marketing Period Days		Probability Adjusted DLOM at Price Volatility		
Mean		10%	40%	70%
	Std Dev	5%	20%	35%
30	15	2.0%	8.3%	14.9%
180	90	5.0%	21.5%	40.0%
365	182.5	7.2%	31.8%	57.4%
730	365	10.3%	46.5%	75.0%
1,825	912.5	16.8%	70.1%	91.6%

Exhibit 13. DLOM Adjusted for Marketing Period and Price Volatility Probabilities				
Marketing Period Days		Probability Adjusted DLOM at Price Volatility		
Mean		10%	40%	70%
	Std Dev	20%	80%	140%
30	60	1.1%	4.7%	8.6%
180	360	2.9%	12.2%	21.0%
365	730	4.1%	17.3%	28.2%
730	1,460	5.9%	23.7%	36.4%
1,825	3,650	9.6%	33.9%	48.1%

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code BVU-2014. The free trial begins July 1, 2014, and runs through July 31, 2014. Please send feedback on this new methodology and tool to andyd@bvresources.com.

Marc Vianello, CPA, ABV, CFF, is managing member of Vianello Forensic Consulting LLC. He can be reached at vianello@vianello.biz. VFC and the author are available for consultation, training, and continuing education seminars regarding the theories and methodologies underlying probability-based DLOM and application of the VFC DLOM Calculator.

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